Math 231 Section B
Summer 2009
Midterm Exam I
July 1, 2009

- You have 50 minutes to complete this exam.
- Show ALL works.
- Please REMOVE hat and sunglasses. Turn off ALL electronic devices.
- Good Luck!

1	2	3	4	5	EC	Total

Name Solutions

UID 000-000-000

Signature Solutions

1. (20 points) Evaluate the following integral:

$$\int xe^{x^2}\cos(x^2)dx$$

(Hint: Don't start IBP right away.)

let $y = x^2$, $\frac{dy}{dx} = 2x$ or $dx = \frac{dy}{2x}$

$$\int xe^{x^2}\cos(x^2)\,dx$$
$$= \int \cancel{x} e^{y}\cos y \frac{dy}{2\cancel{x}}$$
$$= \frac{1}{2}\int e^{y}\cos y\,dy. \quad \cdots (*)$$

$$\int e^{y}\cos y\,dy$$
$$= e^{y}\sin y - \int e^{y}\sin y\,dy$$ let $u = e^{y}$, $dv = \cos y\,dy$; $du = e^{y}dy$, $v = \sin y$

$$= e^{y}\sin y - \left[-e^{y}\cos y + \int e^{y}\cos y\,dy\right]$$ let $u = e^{y}$ $dv = \sin y\,dy$; $du = e^{y}dy$, $v = -\cos y$

$$= e^{y}(\sin y + \cos y) - \int e^{y}\cos y\,dy$$

$$\Rightarrow 2\int e^{y}\cos y\,dy = e^{y}(\sin y + \cos y)$$

$$\Rightarrow \int e^{y}\cos y\,dy = \frac{e^{y}}{2}(\sin y + \cos y) + C'$$

$$\therefore (*) = \frac{1}{4}e^{y}(\sin y + \cos y) + C = \boxed{\frac{1}{4}e^{x^2}(\sin x^2 + \cos x^2) + C}$$

2. (15 points) Evaluate the following integral:

$$\int tan^3\theta sec^3\theta d\theta$$

Let $u = \sec\theta$, $du = \sec\theta \tan\theta \, d\theta$

$$\int \tan^3\theta \sec^3\theta \, d\theta$$

$$= \int \tan^2\theta \sec^2\theta \sec\theta \tan\theta \, d\theta$$

$$= \int (\sec^2\theta - 1) \sec^2\theta \, du$$

$$= \int (u^2 - 1) u^2 \, du$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + c$$

$$= \boxed{\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C}$$

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3. (20 points) Evaluate the following integral:

$$\int_2^4 \frac{2x}{x^2+3x+2}\,dx$$

$$\frac{2x}{x^2+3x+2} = \frac{2x}{(x+2)(x+1)}$$

$$= \frac{A}{x+2} + \frac{B}{x+1}$$

$$2x = A(x+1) + B(x+2)$$

let $x = -1$: $-2 = B$

$x = -2$: $-4 = -A \Rightarrow A = 4$

$$\therefore \quad \frac{2x}{x^2+3x+2} = \frac{4}{x+2} - \frac{2}{x+1}$$

and $$\int_2^4 \frac{2x}{x^2+3x+2}\,dx$$

$$= \left(4\ln|x+2| - 2\ln|x+1|\right)\Big|_2^4$$

$$= 4\ln 6 - 2\ln 5 - 4\ln 4 + 2\ln 3$$

$$= \ln\frac{6^4 \cdot 3^2}{5^2 \cdot 4^4} = \ln\left(\frac{729}{400}\right)$$

- all acceptable -

4. Determine whether the following integrals converge or diverge:

(a) (15 points)

$$\int_1^{\infty} \frac{1}{x^{\frac{3}{2}} + [lnx - cos(7x)]^2} dx$$

(b) (15 points)

$$\int_0^4 \frac{4 - 3sin(e^{7x})}{x^{\frac{7}{4}}} dx$$

(a)

$$\frac{1}{x^{\frac{3}{2}}+\left[\ln x-\cos(7x)\right]^2} \leq \frac{1}{x^{\frac{3}{2}}}$$

$$[\cdots]^2 \geq 0$$

and $\int_1^{\infty} \frac{1}{x^{\frac{3}{2}}} dx < \infty$ ($\int_1^{\infty}$ p-integral w/ $p=\frac{3}{2}>1$)

$\therefore \int \frac{1}{x^{\frac{3}{2}}+\left[\ln x-\cos(7x)\right]^2} dx < \infty$, by comparison test

converges!

(b) $-1 \leq \sin(e^{7x}) \leq 1$

$1 \leq 4-3\sin(e^{7x}) \leq 7$

$$\frac{4-3\sin(e^{7x})}{x^{\frac{7}{4}}} \geq \frac{1}{x^{\frac{7}{4}}}$$

and $\int_0^4 \frac{1}{x^{\frac{7}{4}}} dx = \infty$ ($\int_0^4$ p-integral w/ $p=\frac{7}{4}>1$)

$\Rightarrow \int_0^4 \frac{4-3\sin(e^{7x})}{x^{7/4}} dx = \infty$ diverges!

5. (15 points) Find the limit, if any, of the following sequence: (show ALL works)

$$a_n = ne^{-\frac{n}{2}}$$

Let $f(x) = xe^{-\frac{x}{2}}$, x real's

$(\therefore f(n) = a_n)$

since $\lim_{x \to \infty} xe^{-\frac{x}{2}}$

$= \lim_{x \to \infty} \frac{x}{e^{\frac{x}{2}}}$

$= \lim_{x \to \infty} \frac{1}{\frac{1}{2}e^{\frac{x}{2}}}$

$= 0$

and all a_n's lie on $f(x)$

$\therefore$ $a_n \to 0$ as well.

(Extra Credit - 10 points) Evaluate the following integral:

$$\int \frac{1}{x^2 - 6x + 13} dx$$

Note: $x^2 - 6x + 13$

$= x^2 - 6x + 9 + 4$

$= (x-3)^2 + 4$

$= (x-3)^2 + 2^2$

$$\int \frac{1}{x^2 - 6x + 13} dx$$

$$= \int \frac{1}{(x-3)^2 + 2^2} dx$$

$$= \int \frac{\cancel{2} \sec^2\theta}{2^{\cancel{2}}(\tan^2\theta + 1)} d\theta$$

Let
$x - 3 = 2\tan\theta$
$dx = 2\sec^2\theta \, d\theta$

$$= \frac{1}{2}\int \frac{\cancel{\sec^2\theta}}{\cancel{\sec^2\theta}} d\theta$$

$$= \frac{1}{2}\theta + C$$

$$= \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C}$$

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