Math 231 Section B
Summer 2009
Midterm Exam III
August 5, 2009

- You have 60 minutes to complete this exam.
- Show ALL works.
- Please REMOVE hat and sunglasses. Turn off ALL electronic devices.
- Good Luck!

1	2	3	4	5	EC	Total

Name ______________________

UID ________________

Signature_________________________

1. (15 points) Find the first three terms of the Maclaurin series of:

$$\sqrt[3]{1-x^2}$$

$$\sqrt[3]{1-x^2} = (1-x^2)^{\frac{1}{3}} = (1+(-x^2))^{\frac{1}{3}}$$
$$= \sum_{k=0}^{\infty} \binom{\frac{1}{3}}{k} (-x^2)^k$$

$k=0$: $\binom{\frac{1}{3}}{0} (-x^2)^0 = 1$

$k=1$: $\binom{\frac{1}{3}}{1} (-x^2)^1 = \frac{1}{3}(-x^2) = -\frac{x^2}{3}$

$k=2$: $\binom{\frac{1}{3}}{2} (-x^2)^2 = \frac{(\frac{1}{3})(\frac{1}{3}-1)}{2} (-x^2)^2$

$$= \frac{\frac{1}{3}(-\frac{2}{3})}{2} (x^4) = -\frac{x^4}{9}$$

5 pts each

2. Consider the function $f(x) = sin(-x)$

(a) (5 points) Find its Taylor series about $c = 0$

(b) (10 points) Write down expression of remainder $R_n(x)$, be sure to specify all the terms.

(c) (15 points) Start by taking a few derivatives of f, observe their patterns, and show that its Taylor series is indeed $f(x)$ for all x

ⓐ $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$

$$\therefore \sin(-x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (-x)^{2k+1}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \overbrace{(-1)^{2k+1}}^{=-1} x^{2k+1}$$

$$= -\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = -\sin x.$$

ⓑ $R_n(x) = \frac{f^{(n+1)}(z)}{(2n+3)!}(-x)^{2(n+1)+1} = \frac{f^{(n+1)}(z)}{(n+1)!}(-x)^{2n+3}$ 7 pts

where z is between 1 & x. ~~3 pts~~ 3 pts

ⓒ $f^{(0)}(z) = \sin(-z)$

$f^{(1)}(z) = -\cos(-z)$

$f^{(2)}(z) = -\sin(-z)$

$f^{(3)}(z) = \cos(-z)$

$\vdots$

we see that $f^{(n+1)}(z)$ is one of the followings: $\pm \sin z$, $\pm \cos z$.

in any case, $|f^{(n+1)}(z)| \le 1$

$$\therefore \quad |R_n(x)| = \frac{|f^{(n+1)}(z)|}{(n+1)!}|x|^{2n+3}$$

$$\le \frac{|x|^{2n+3}}{(n+1)!} \xrightarrow{n\to\infty} 0 \text{ for ALL } x$$

(factorial at bottom exponential on top)

(OR consider

$$\sum_{n=1}^{\infty} \frac{|x|^{2n+3}}{(n+1)!},$$

it converges by ratio test, $\therefore$ kTH term test: $\frac{|x|^{2n+3}}{(n+1)!} \xrightarrow{n\to\infty} 0$

3. Consider a circle, centered at (1,1) with radius 2.

(a) (10 points) Construct a parametrized equation of this circle.

(b) (10 points) Determine locations of horizontal tangent line to the circle. Indicate values of t at those points.

circle centered @ (1,1) w/ radius 2

has rectangular equation

$$(x-1)^2 + (y-1)^2 = 2^2$$

(a) one possible parametrization is

$$x = 1 + 2\cos t$$
$$y = 1 + 2\sin t$$

(b) $y'(t) = 2\cos t = 0$ @ $t = \pi/2,\ 3\pi/2$

b $x'(\frac{\pi}{2}) = -2\sin(\frac{\pi}{2}) \neq 0$

$x'(\frac{3\pi}{2}) = -2\sin(\frac{3\pi}{2}) \neq 0$

$\therefore$ horizontal tangent lines

when $t = \pi/2,\ 3\pi/2$, which

correspond to $(x(\pi/2), y(\pi/2))$

$= (3,1)$

$(x(3\pi/2), y(3\pi/2))$

$= (-1,1)$

4. Consider the following parametrized equation:

$$\begin{cases} x(t) = \frac{t^2}{2} + 2 \\ y(t) = \frac{1}{6}(4t+4)^{\frac{3}{2}} \end{cases}$$

$$0 \le t \le 1$$

(a) (10 points) Find the arclength within the given period of time.

(b) (5 points) Set up, but do NOT integrate, the expression for surface area generated by this piece of curve after revolving around x axis.

(a)

$x' = t$

$y' = \frac{3}{\cancel{2}} \cdot \frac{1}{\cancel{6}}(4t+4)^{\frac{1}{2}} \cdot \cancel{(4)} = \sqrt{4t+4}$

$\therefore \quad l = \int_0^1 \sqrt{t^2 + (\sqrt{4t+4})^2}\, dt$

$= \int_0^1 \sqrt{t^2+4t+4}\, dt$

$= \int_0^1 \sqrt{(t+2)^2}\, dt = \int_0^1 (t+2)\, dt$

$= \left(\frac{t^2}{2} + 2t\right)\Big|_0^1 = \frac{5}{2}$ //

(b)

$A = 2\pi \int_0^1 \frac{1}{6}(4t+4)^{\frac{3}{2}}(t+2)\, dt$

//

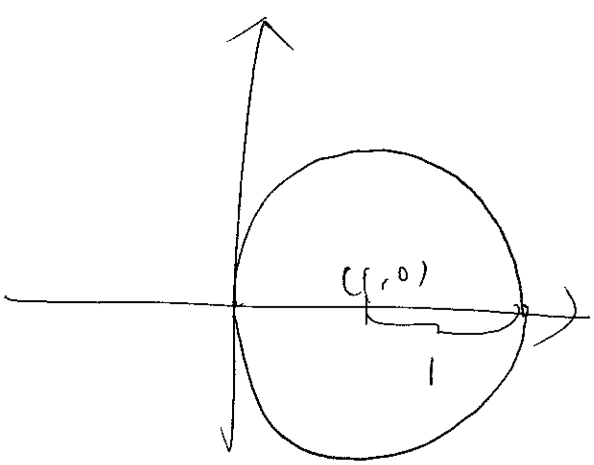
5. (20 points) Sketch the following polar equation, be sure to indicate relevent coordinates.

$$r = 2cos\theta$$

$$\left.\begin{aligned} r^2 &= 2r\cos\theta \\ x^2+y^2 &= 2x \\ x^2-2x+y^2 &= 0 \\ x^2-2x+1+y^2 &= 1 \\ (x-1)^2+y^2 &= 1 \end{aligned}\right\} \text{5 pts}$$

$\therefore$ circle centered @ (1, 0) w/ radius 1

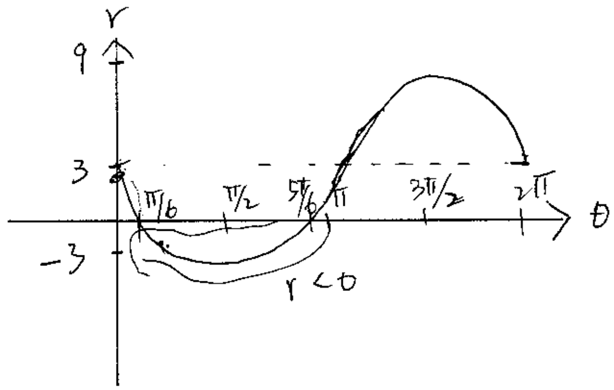
5 pts for recognizing this fact



} 10 pts

(Extra Credit - 10 points) Sketch the following polar equation, be sure to indicate relevent coordinates:

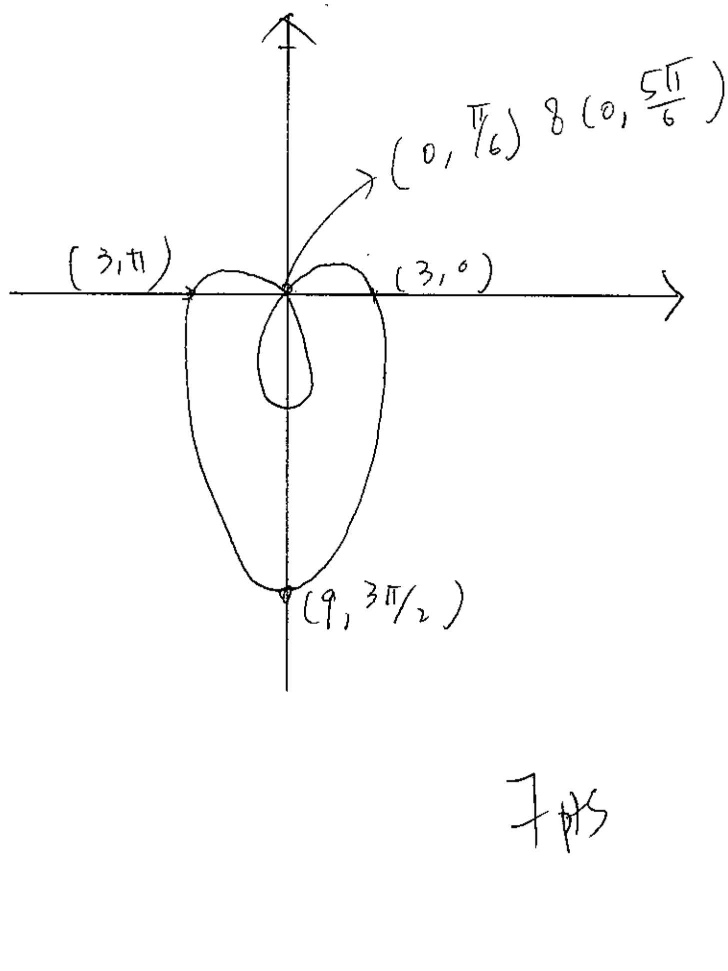
$$r = 3 - 6 sin\theta$$



r = 0 when $\sin\theta = \frac{1}{2}$

ie $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

3 pts



7 pts