HW13 (Graded) Solutions

§9.3 2, 8, 12, 22, 24

(2) $\begin{cases} x = 1 - 2\cos t \\ y = 2 + 3\sin t \end{cases}$

one cycle / loop goes from $t=0$ to $t=2\pi$

$x' = 2\sin t$ & $y' = 3\cos t$

$(x')^2 + (y')^2 = 4\sin^2 t + 9\cos^2 t = 4\sin^2 t + 4\cos^2 t + 5\cos^2 t$
$= 4 + 5\cos^2 t$

$\therefore \quad l = \int_0^{2\pi} \sqrt{4 + 5\cos^2 t}\, dt$ //

(8) $\begin{cases} x = t^2\cos t \\ y = t^2 \sin t \end{cases}$, $-1 \le t \le 1$

$x' = 2t\cos t - t^2 \sin t$
$y' = 2t\sin t + t^2\cos t$

$(x')^2 + (y')^2 = 4t^2 + t^4 - \cancel{4t^3\sin t\cos t + 4t^3 \sin t\cos t}$

$\therefore \quad l = \int_{-1}^{1} \sqrt{4t^2 + t^4}\, dt$

$= \int_{-1}^{1} |t|\sqrt{4+t^2}\, dt = 2\int_0^1 t\sqrt{\quad}\, dt$

$u = 4+t^2 \quad = \cancel{2}\int_4^5 \cancel{t}\sqrt{u}\,\frac{du}{\cancel{2t}}$

$= \frac{2}{3}\sqrt{u^3}\Big|_4^5 = \frac{2}{3}\left(\sqrt{125} - 8\right)$ //

(12) $\begin{cases} x = \sin t \\ y = \sin\sqrt{2}t \end{cases}$ $0 \le t \le \pi$ $\rightarrow$ $\begin{cases} x' = \cos t \\ y' = \sqrt{2}\cos\sqrt{2}t \end{cases}$

$l = \int_0^{\pi} \left(\cos^2 t + 2\cos^2\sqrt{2}t\right) dt$ //

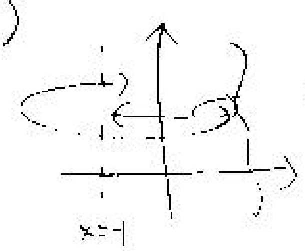
(22) $\begin{cases} x = t^2-1 \\ y = t^3-4t \end{cases}$ $\quad 0 \le t \le 2.$ $\quad$ about x-axis

$\begin{cases} x' = 2t \\ y' = 3t^2-4 \end{cases}$ $\quad dl = \sqrt{(x')^2+(y')^2}\,dt = \sqrt{4t^2+(3t^2-4)^2}\,dt$

$$A = 2\pi \int_0^2 (t^3-4t)\sqrt{4t^2+9t^4-24t^2+16}\,dt$$

$$= 2\pi \int_0^2 (t^3-4t)\sqrt{9t^4-20t^2+16}\,dt \quad //$$

(24)



, radius $= 1+x(t)$

$\begin{cases} x(t) = t^2-1 \\ y(t) = t^3-4t \end{cases}$ $\quad -2 \le t \le 0$

$x' = 2t \quad \& \quad y' = 3t^2-4$

$$dl = \sqrt{(2t)^2 + (3t^2-4)^2}\,dt$$

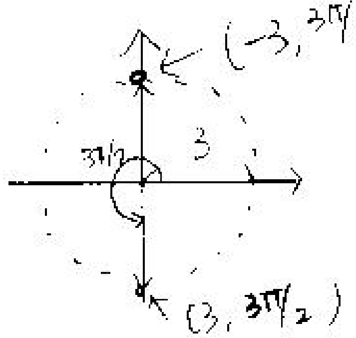
$$= \sqrt{9t^4+20t^2+16}\,dt$$

$$A = 2\pi \int_{-2}^{0} (1+x(t))\,dl$$

$$= 2\pi \int_{-2}^{0} t^2\sqrt{9t^4+20t^2+16}\,dt \quad //$$

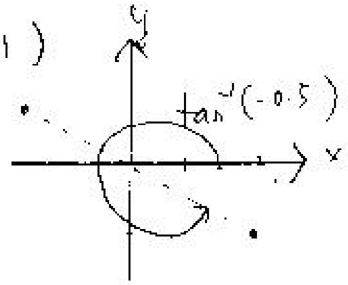
§9.4 : 4, 10, 12, 18, 20, 22, 26, 30, 34, 42, 48, 58, 60

(4) $(-3, 3\pi/2)$



Rectangular Coordinate
$(0, -3)$
$x = -3\cos 3\pi/2 = 0$
$y = -3\sin 3\pi/2 = 3$

(10) $(x, y) = (2, -1)$



$r = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\theta = \tan^{-1}\left(\frac{-1}{2}\right) \sim 5.81$ (rad)

$\therefore \left(\sqrt{5}, \tan^{-1}\left(\frac{-1}{2}\right)\right)$ to be in IV quadrant

other representations include :

$$\left(\sqrt{5}, \tan^{-1}\left(\tfrac{-1}{2}\right) + 2n\pi\right)$$

$$\left(-\sqrt{5}, \tan^{-1}\left(\tfrac{-1}{2}\right) \pm (2n+1)\pi\right).$$

(12) $(x, y) = (-2, -\sqrt{5})$
(III quadrant)

$r = \sqrt{(-2)^2 + (-\sqrt{5})^2} = \sqrt{29}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{5}}{-2}\right) = 3.98$ rad
(to be @ III quadrant)

$\therefore (\sqrt{29}, 3.98)$

other representations include :

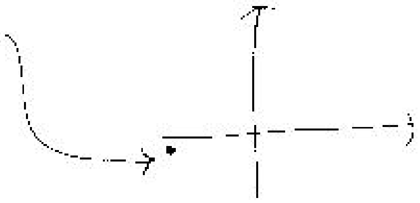
$$(\sqrt{29}, 3.98 \pm 2n\pi)$$

$$(-\sqrt{29}, 3.98 \pm (2n+1)\pi)$$

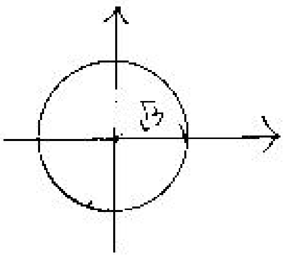
//

(18) $(r,\theta) = (-3, 1)$

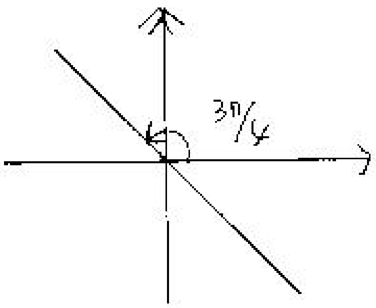
$x = r\cos\theta = -3\cos 1$
$y = r\sin\theta = -3\sin 1$



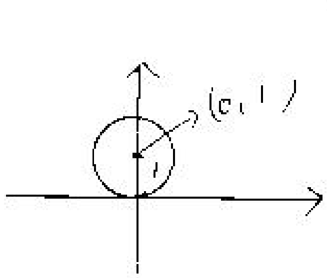
(20) $r = \sqrt{3} \Rightarrow$ circle of radius $\sqrt{3}$ centered @ origin



(22) $\theta = 3\pi/4 \Rightarrow$ a line of slope $\tan 3\pi/4 \approx -1$



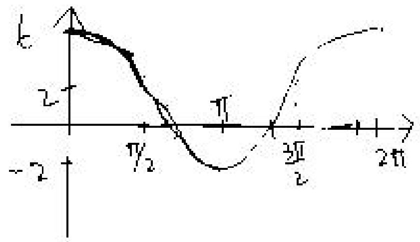
(26) $r = 2\sin\theta$
$r^2 = 2r\sin\theta$
$x^2 + y^2 = 2y \Rightarrow x^2 + \underbrace{y^2 - 2y}_{} = 0$
$x^2 + (y-1)^2 = 1$ circle centered @ $(0,1)$ w/ radius 1

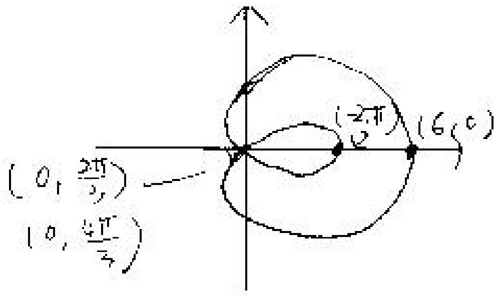


(30) $r = \sin(2\theta)$

see example 4.12 (not needed for exam)

(34) $r = 2 + 4\cos\theta$, $r = 0$ @ $\cos\theta = \frac{-1}{2}$, i.e. $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$





(42) $r = \cos\theta + \sin 2\theta$
$= \cos\theta + 2\sin\theta\cos\theta$
$= \cos\theta(1 + 2\sin\theta)$
$r^3 = r\cos\theta(r + 2r\sin\theta)$
$(x^2+y^2)^{\frac{3}{2}} = x($

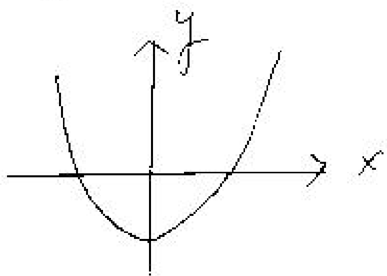
(42) to be uploaded soon.

(48) $r = \frac{3}{1-\sin\theta} \Rightarrow r - r\sin\theta = 3$

$\sqrt{x^2+y^2} = 3 + y$

OR $x^2 + \cancel{y^2} = (3+y)^2 = 9 + 6y + \cancel{y^2}$

$\Rightarrow y = \frac{x^2-9}{6}$ parabola



(59) $x^2+y^2=9 \Rightarrow r=3$
(circle of radius 3)

(60) $x^2+y^2=x$
$r^2 = r\cos\theta$, OR $r=\cos\theta$ if $r\neq 0$