HW#1 Solutions ‹Optional›

(2) $\int x \sin(4x)\, dx$

$= -\frac{1}{4}\cos(4x) + \frac{1}{4}\int \cos(4x)\,dx$ $\quad u = x\ ;\ dv = \sin(4x)\,dx$
$\quad du = dx\ ;\ v = -\frac{1}{4}\cos(4x)$

$= \boxed{-\frac{1}{4}\cos(4x) + \frac{1}{16}\sin(4x) + C}$ ANS

(8) $\int x^2 e^{x^3}\, dx$

$= \frac{1}{3}\int e^u\, du$ $\quad u = x^3, \quad dx = du/3x^2$

$= \boxed{\frac{1}{3} e^{x^3} + C}$ ANS Note: no ibp needed.

(11) $\int \cos x \cos(2x)\, dx$

$= \frac{1}{2}\cos x \sin(2x) + \frac{1}{2}\int \sin x \sin(2x)\,dx$ $\quad u = \cos x\ ;\ dv = \cos(2x)\,dx$
$\quad du = -\sin x\,dx\ ;\ v = \frac{1}{2}\sin(2x)$

$= \frac{1}{2}\cos x \sin(2x)$
$+ \frac{1}{2}\left[-\frac{1}{2}\sin x \cos(2x) + \frac{1}{2}\int \cos x \cos(2x)\,dx\right]$ $\quad u = \sin x\ ;\ dv = \sin(2x)\,dx$
$\quad du = \cos x\,dx\ ;\ v = -\frac{1}{2}\cos(2x)$

$= \frac{1}{2}\cos x \sin(2x) - \frac{1}{4}\sin x \cos(2x) + \underline{\frac{1}{4}\int \cos x \cos(2x)\,dx}$

$= \int \underline{\cos x \cos(2x)}\,dx$ ← combine

$\frac{3}{4}\int \cos x \cos(2x)\,dx = \frac{1}{2}\cos x \sin(2x) - \frac{1}{4}\sin x \cos(2x) + C'$

OR $\int \cos x \cos(2x)\,dx = \boxed{\frac{2}{3}\cos x \sin(2x) - \frac{1}{3}\sin x \cos(2x) + C}$

ANS

(18) $\int x \sin x^2 \, dx$

$= \frac{1}{2} \int \cancel{x} \sin u \frac{du}{\cancel{x}}$ $\quad u = x^2$, $dx = du/2x$

$= -\frac{1}{2} \cos u + C = \boxed{-\frac{1}{2} \cos x^2 + C.}$ ANS.

(24) $\int_0^1 x \cos(2\pi x) \, dx$

$= \cancel{\frac{x}{2\pi} \sin(2\pi x) \Big|_0^1} + \frac{1}{(4\pi)^2} \cos(2\pi x) \Big|_0^1 = 0$

(31) $\int \sin(\ln x) \, dx$

$= \int e^y \sin y \, dy$ $\quad y = \ln x$, $dx = x\,dy = e^y dy$

see $\int e^x \cos x \, dx$ $\quad \overset{\Downarrow}{=} \frac{e^y}{2} (\sin y - \cos y) + C$

$= \boxed{\frac{x}{2} [\sin(\ln x) - \cos(\ln x)] + C}$ ANS.

(40) Reduction Formula for Sine:

SHOW. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

$\int \sin^n x\,dx$

$= -\sin^{n-1}\cos x + (n-1)\int \sin^{n-2}x\cos x^2\,dx$ $\quad u=\sin^{n-1}x$; $dv=\sin x\,dx$; $du=(n-1)\sin^{n-2}x\cos x$; $v=-\cos x$

$\cos^2 x = 1-\sin^2 x$

$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\,dx$

$= -\sin^{n-1}x\cos x + (n-1)\int (\sin^{n-2}x - \sin^n x)\,dx$

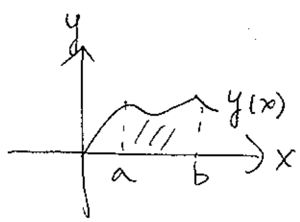
$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx \underline{-(n-1)\int \sin^n x\,dx}$

$= \underline{\int \sin^n x\,dx}$ combine

$n\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx$

OR $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$ //

(62) in general



volume of solid formed by rotating $y(x)$ around x-axis is given by

$$\int_a^b \pi y^2\,dx$$

Here $y(x) = x\sqrt{\sin x}$, $a=0$, $b=\pi$

$\therefore\ v = \pi\int_0^\pi x^2\sin x\,dx$

$= \pi\left[-x^2\cos x\Big|_0^\pi + 2\int_0^\pi x\cos x\,dx\right]$ $\quad u=x^2$, $dv=\sin x\,dx$; $du=2x\,dx$, $v=-\cos x$

$= \pi\left[\pi^2 + 2\left[x\sin x + \cos x\right]_0^\pi\right]$

$= \pi\left[\pi^2 - 4\right]$ // ANS

(5)
$$\int_0^{\pi/2} \cos^2 x \sin x \, dx$$
$$= -\int_{\cos 0}^{\cos \pi/2} u^2 \cancel{\sin x} \frac{du}{\cancel{\sin x}}$$
$u = \cos x$
$dx = du/-\sin x$
$$= -\int_1^0 u^2 \, du$$
$$= \frac{u^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

(9)
$$\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$$
$$= \int_0^{\pi/4} \tan^4 x \sec^2 x \sec^2 x \, dx$$
$$= \int_0^1 u^4(\tan^2 x + 1) \, du$$
$u = \tan x$
$du = \sec^2 x \, dx$
$$= \int_0^1 u^4(u^2+1) \, du$$
$$= \int_0^1 (u^6 + u^4) \, du = \frac{1}{7} + \frac{1}{5} = \boxed{\frac{12}{35}}$$

(31) $u = \tan x$, $du = \sec^2 x \, dx$
$$\int \tan x \sec^4 x \, dx$$
$$= \int u \sec^2 x \sec^2 x \, dx$$
$$= \int u(u^2+1) \, du = \int (u^3 + u) \, du = \frac{u^4}{4} + \frac{u^2}{2} + C$$
$$= \boxed{\frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + C}$$

$u = \sec x$, $du = \sec x \tan x \, dx$
$$\int \tan x \sec^4 x \, dx$$
$$= \int \sec^3 x \sec x \tan x \, dx$$
$$= \int u^3 \, du = \frac{u^4}{4} = \boxed{\frac{\sec^4 x}{4} + C}$$

Show they're the same!