HW#2 Solutions (Graded)

§ 6.3 18, 22, 24, 33, 34

(18) $\displaystyle\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

$x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$16 - x^2 = 16\cos^2\theta$

$(16-x^2)^{1/2} = 4\cos\theta$

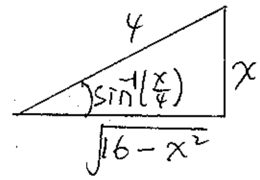
$\displaystyle = \int \frac{1}{16\sin^2\theta \cdot \cancel{4\cos\theta}}\,\cancel{4\cos\theta}\, d\theta$

$\displaystyle = 16\int \csc^2\theta\, d\theta$

$= -16\cot\theta + C$

$= -16\cot\left(\sin^{-1}\left(\frac{x}{4}\right)\right) + C$

$\displaystyle = \boxed{-16\,\frac{\sqrt{16-x^2}}{x} + C}$ // ANS.



(22) $\displaystyle\int x^3\sqrt{x^2-1}\,dx$

$x = \sec\theta \qquad x^2 - 1 = \tan^2\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$\displaystyle = \int \sec^3\theta \tan\theta \sec\theta \tan\theta\, d\theta$

$\displaystyle = \int \sec^4\theta \tan^2\theta\, d\theta$

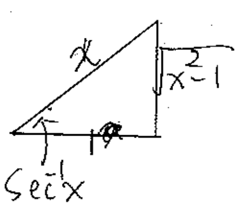
$\displaystyle = \int \sec^2\theta \tan^2\theta \sec^2\theta\, d\theta$

$u = \tan\theta,\ du = \sec^2\theta\, d\theta$

$\displaystyle = \int (\tan^2\theta - 1)\, u^2\, du = \int u^2(u^2-1)\, du$

$\displaystyle = \int (u^4 - u^2)\, du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$\displaystyle = \frac{\tan^5\theta}{5} - \frac{\tan^3\theta}{3} + C$



$\displaystyle = \frac{1}{5}\left(\tan\left(\sec^{-1}x\right)\right)^5 - \frac{1}{3}\left(\tan\left(\sec^{-1}x\right)\right)^3 + C$

$\displaystyle = \boxed{\frac{1}{5}\left(\sqrt{x^2-1}\right)^5 - \frac{1}{3}\left(\sqrt{x^2-1}\right)^3 + C}$ // ANS.

(24) $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,dx$

$\displaystyle= \frac{1}{2}\int \frac{\cancel{x}}{\sqrt{u}}\,\frac{du}{\cancel{x}}$ $\qquad u = x^2-4$, $\;dx = du/2x$

$= \sqrt{u} = \boxed{\sqrt{x^2-4} + C}$ // ANS.

(33) SHOW, for $n>1$,

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

Let $u = \sec^{n-2}x$, $\quad dv = \sec^2 x\,dx$

$du = (n-2)\sec^{n-3}x \cdot \text{sex}\tan x\,dx$; $\quad v = \tan x$

$= (n-2)\sec^{n-2}x\tan x\,dx$

Using IBP, we have

$$\int \sec^n x\,dx$$

$$= \tan x\sec^{n-2}x - (n-2)\int \sec^{n-2}x\tan^2 x\,dx$$

$$= \tan x\sec^{n-2}x - (n-2)\int \sec^{n-2}x\,(\sec^2 x - 1)\,dx$$

$$= \tan x\sec^{n-2}x - (n-2)\int \sec^n x\,dx - (n-2)\int \sec^{n-2}x\,dx$$

$$= \int \sec^n x\,dx \leftarrow \text{combine}$$

$$(n-1)\int \sec^n x\,dx = \sec^{n-2}x\tan x - (n-2)\int \sec^{n-2}x\,dx$$

OR. $$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x - \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$ //

(34) ⓐ $\int \sec^3 x \, dx$

(33) w/ n=3 $= \frac{1}{2}\sec x \tan x + \frac{1}{2}\int \sec x \, dx$

ex 3.8 $= \boxed{\frac{1}{2}\sec x \tan x + \ln|\sec x + \tan x| + C.}$ // ANS

ⓑ $\int \sec^4 x \, dx$

$\overset{n=4}{=} \frac{1}{3}\sec^2 x \tan x + \frac{2}{3}\int \sec^2 x \, dx$

$= \boxed{\frac{1}{3}\sec^2 x \tan x + \frac{2}{3}\tan x + C.}$ // ANS.

ⓒ $\int \sec^5 x \, dx$

$\overset{n=5}{=} \frac{1}{4}\sec^3 x \tan x + \frac{3}{4}\int \sec^3 x \, dx$ part ⓐ //.

§6-4 2, 16, 26.

(2) $\frac{5x-2}{x^2-4} = \frac{5x-2}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$

$5x-2 = A(x-2) + B(x+2)$

$x=2: \quad 8 = 4B \Rightarrow B = 2$

$x=-2: \quad -12 = -4A \Rightarrow A = 3$

$\therefore \int \frac{5x-2}{x^2-4} dx = \int \frac{3}{x+2} dx + \int \frac{2}{x-2} dx$

$= \boxed{3\ln|x+2| + 2\ln|x-2| + C}$ //

(1b) $\dfrac{1}{x^3+4x} = \dfrac{1}{x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$

$$1 = A(x^2+4) + (Bx+C)x$$
$$= (A+B)x^2 + Cx + 4A$$

$\Rightarrow A = 1/4,\ C = 0,\ B = -1/4$

$$\int \frac{1}{x^3+4x}\,dx = \frac{1}{4}\ln|x| - \frac{1}{4}\int \frac{x}{x^2+4}\,dx$$
$$= \boxed{\frac{1}{4}\ln|x| - \frac{1}{8}\ln|x^2+4| + C}$$ //

(2b) since $\deg(x^3-2x) > \deg(2x^2-3x+2)$
long division required!

$$\begin{array}{r|l} & \frac{x}{2} + \frac{3}{4} \\ 2x^2-3x+2 & x^3 + 0x^2 - 2x + 0 \\ & x^3 - \frac{3}{2}x^2 + x \\ \hline & \frac{3}{2}x^2 - 3x + 0 \\ & \frac{3}{2}x^2 - \frac{9}{4}x + \frac{3}{2} \\ \hline & -\frac{3}{4}x - \frac{3}{2} \end{array}$$

ie $\dfrac{x^3-2x}{2x^2-3x+2} =$

$$= \frac{x}{2} + \frac{3}{4} + \frac{-\frac{3}{4}x - \frac{3}{2}}{2x^2-3x+2}$$

To integrate, the only troublesome one will be $\displaystyle\int \frac{1}{2x^2-3x+2}\,dx$ (note: $2x^2-3x+2$ is irreducible)

trick: $2x^2-3x+2 = 2\left(x^2 - \frac{3}{2}x + 1\right)$
$$= 2\left(x^2 - 2\cdot\frac{3}{4}x + \frac{9}{16} + \frac{7}{16}\right)$$
$$= 2\left[\left(x-\frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2\right]$$

$$\int \frac{1}{2x^2-3x+2}\,dx = \frac{1}{2}\int \frac{1}{\left(x-\frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}\,dx$$

let $x = \frac{3}{4} + \frac{\sqrt{7}}{4}\tan\theta$
$dx = \frac{\sqrt{7}}{4}\sec^2\theta\,d\theta$

$$= \frac{1}{2}\int \frac{\left(\frac{\sqrt{7}}{4}\right)\cancel{\sec^2\theta}}{\left(\frac{\sqrt{7}}{4}\right)^2\cancel{(\tan^2\theta+1)}}\,d\theta$$

$$= \frac{1}{2}\cdot\frac{16}{7}\,\theta + C = \frac{8}{7}\theta + C = \frac{8}{7}\tan^{-1}x + C$$ //