HW#2 Solutions (Optional)

§6-3 : 11, 16, 30, 31, 36.

(11) $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$

$= \int_0^1 u^4 \sec^2 x \sec^2 x \, dx$ $\quad u = \tan x$, $du = \sec^2 x \, dx$

$= \int_0^1 u^4 (\tan^2 x + 1) \, du$

$= \int_0^1 u^4 (u^2+1) \, du = \int_0^1 (u^6 + u^4) \, du = \frac{1}{7} + \frac{1}{5} = \boxed{\frac{12}{35}}$ //

(16) $\int_{\pi/4}^{\pi/2} \cot^2 x \csc^4 x \, dx$

$= \int_1^0 u^2 \csc^2 x \csc^2 x \, dx$ $\quad u = \cot x$, $du = -\csc^2 x \, dx$

$= -\int_1^0 u^2 (\cot^2 x + 1) \, du$

$= \int_0^1 u^2 (u^2+1) \, du$

$= \int_0^1 (u^4 + u^2) \, du = \frac{1}{5} + \frac{1}{3} = \boxed{\frac{8}{15}}$ // ANS

(30) $\int_0^2 x^2 \sqrt{x^2+9} \, dx$

$= \int_0^{\tan^{-1}(\frac{2}{3})} 9\tan^2\theta \cdot 3\sec\theta \cdot 3\sec^2\theta \, d\theta$ $\quad x = 3\tan\theta$, $dx = 3\sec^2\theta \, d\theta$, $x^2+9 = 9\sec^2\theta$

$= 81 \int_0^{\tan^{-1}(\frac{2}{3})} \tan^2\theta \sec^3\theta \, d\theta$

$= 81 \int_0^{\tan^{-1}(\frac{2}{3})} (\sec^2\theta - 1) \sec^3\theta \, d\theta$

$= 81 \int_0^{\tan^{-1}(\frac{2}{3})} \sec^5\theta \, d\theta - 81 \int_0^{\tan^{-1}(\frac{2}{3})} \sec^3\theta \, d\theta$

$= \dots$ reduction formula in (34)

(36) $\frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$ $\quad x = a\sec\theta$, $dx = a\sec\theta\tan\theta\,d\theta$, $a^2-x^2 = a^2\tan^2\theta$

$= \frac{4b}{a}\int_{\sec^{-1}0}^{\sec^{-1}1} a\tan\theta \cdot a\sec\theta\tan\theta\,d\theta$

$= 4ab\int_{\sec^{-1}0}^{\sec^{-1}1} \tan^2\theta\sec\theta\,d\theta$

$= 4ab\int_{\pi/2}$

(36) $\frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$ $\quad x = a\sin\theta$, $dx = a\cos\theta\,d\theta$, $a^2-x^2 = a^2\cos^2\theta$

$= 4ba\int_{\sin^{-1}0}^{\sin^{-1}1} \cos^2\theta\,d\theta$

$= 4ba\int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\,d\theta$

$= 2ab\int_0^{\pi/2} (1+\cos(2\theta))\,d\theta$

$= 2ab\left(\theta + \frac{\sin(2\theta)}{2}\Big|_0^{\pi/2}\right) = ab\pi$ // ANS

§6-4 : 9, 19, 30

(9) $\frac{5x-23}{6x^2-11x-7} = \frac{5x-23}{(2x+1)(3x-7)} = \frac{A}{2x+1} + \frac{B}{3x-7}$

$5x-23 = A(3x-7) + B(2x+1)$

$x = \frac{7}{3}$: $-\frac{34}{3} = \frac{17}{3}B \Rightarrow B = -2$

$x = -\frac{1}{2}$: $-\frac{51}{2} = -\frac{17}{2}A \Rightarrow A = 3$

$\int \frac{5x-23}{6x^2-11x-7}\,dx = 3\int \frac{1}{2x+1}\,dx - 2\int \frac{1}{3x-7}\,dx$

$= \boxed{\frac{3}{2}\ln|2x+1| - \frac{2}{3}\ln|3x-7| + C}$ // ANS

(19) $\deg(4x^2-7x-17) = \deg(6x^2-11x-10)$
$\therefore$ require long division

$$\begin{array}{r|l} & \frac{2}{3} \\ 6x^2-11x-10 & 4x^2-7x-17 \\ & \underline{\quad -\frac{22}{3}x - \frac{20}{3}} \\ & \quad \frac{1}{3}x - \frac{31}{3} \end{array}$$

$$\therefore \frac{4x^2-7x-17}{6x^2-11x-10} = \frac{2}{3} + \frac{\frac{1}{3}x - \frac{31}{3}}{6x^2-11x-10}$$

$$\frac{\frac{1}{3}x - \frac{31}{3}}{6x^2-11x-10} = \frac{A}{2x-5} + \frac{B}{3x+2}$$

$$\frac{1}{3}x - \frac{31}{3} = A(3x+2) + B(2x-5)$$

$x = -\frac{2}{3}$: $\quad -\frac{95}{9} = -\frac{19}{3}B \Rightarrow B = \frac{5}{3}$

$x = \frac{5}{2}$: $\quad -\frac{57}{6} = \frac{19}{2}A \Rightarrow A =$

$$\therefore \int \frac{4x^2-7x-17}{6x^2-11x-10}\,dx = \boxed{\frac{2}{3}x - \frac{1}{2}\ln|2x-5| + \frac{5}{3}\ln|3x+2| + C}$$

ANS

(30) Again, long division required.

$$\begin{array}{r|l} & 2x \\ x^3+0x^2+4x+0 & 2x^4+0x^3+9x^2+x-4 \\ & \underline{2x^4+0x^3+8x^2+0x} \\ & \qquad\qquad x^2+x-4 \end{array}$$

$$\frac{2x^4+9x^2+x-4}{x^3+4x} = 2x + \underline{\qquad\qquad}$$

$$= 2x + \frac{x^2+x-4}{x(x^2+4)}$$

$$\frac{x^2+x-4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$x^2+x-4 = A(x^2+4) + x(Bx+C)$$
$$= (A+B)x^2 + Cx + 4A$$

$\Rightarrow A = -1,\ C = 1,\ B = 2 \longrightarrow$ take $\int$. (note $\int \frac{1}{x^2+4}dx \overset{2u=x}{=} 2\int \frac{1}{4u^2+4}du$ ...