HW #3 (Graded)
§6-4 : 18, 34

(18) $\frac{3x+7}{x^4-16} = \frac{3x+7}{(x^2+4)(x+2)(x-2)}$
$= \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}$

OR $3x+7 = A(x-2)(x^2+4) + B(x+2)(x^2+4) + (Cx+D)(x-2)(x+4)$
let $x=2$: $13 = 32B$ $\Rightarrow$ get $B = 13/32$.
let $x=-2$: $1 = -32A$ $\Rightarrow$ get $A = -1/32$.

Compare coefficient of ...
x^3: $0 = A+B+C$
x^0: $7 = -8A+8B-8D$
$\Rightarrow \begin{cases} C = -\frac{12}{32} \\ D = -\frac{7}{16} \end{cases}$ ... and $\int$

(34) $\frac{2x^3-x^2}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$

$2x^3-x^2 = (Ax+B)(x^2+1) + Cx+D$
$= Ax^3+Bx^2+Ax+B+Cx+D$
$= Ax^3+Bx^2+(A+C)x+B+D$
$\Rightarrow$ $A=2$, $B=-1$, $A+C=0 \Rightarrow C=-2$
$B+D=0 \Rightarrow B=1$. ... and $\int$

, we need to worry about
$\int \frac{1}{(x^2+1)^2}\,dx$ ..
let $x = \tan\theta$, $x^2+1 = \tan^2\theta+1 = \sec^2\theta$
$dx = \sec^2\theta\, d\theta$

$\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \int \frac{d\theta}{\sec^2\theta} = \underline{\int \cos^2\theta\, d\theta}$

we know this !!

§6-6 2, 6, 12, 18, 26, 34.

(2) Not improper, $x^{2/5}$ is never 0 on $[1,2]$.

(6) Improper ... limit of integration is not bounded.

(12) $\int_1^5 \frac{2}{\sqrt{5-x}}\,dx = \lim_{R\to 5^-} \int_1^R \frac{2}{\sqrt{5-x}}\,dx$

$u = 5-x$
$dx = -du$

$= \lim_{R\to 5^-} -\int_4^{5-R} \frac{2}{\sqrt{u}}\,du$

$= \lim_{R\to 5^-} -4\sqrt{u}\Big|_4^{5-R}$

$= \lim_{R\to 5^-} \left(8 - 4\sqrt{5-R}\right) = 8 < \infty$ //

(18) $\int_1^\infty x^2 e^{-2x}\,dx = \lim_{R\to\infty} \int_1^R x^2 e^{-2x}\,dx$

$\int x^2 e^{-2x}\,dx$

$= -\frac{x^2e^{-2x}}{2} + \int x e^{-2x}\,dx$

$u = x^2$; $dv = e^{-2x}dx$
$du = 2x\,dx$ $v = e^{-2x}/-2$

$= -\frac{1}{2}x^2e^{-2x} + \left[-\frac{xe^{-2x}}{2} + \int \frac{e^{-2x}}{2}\,dx\right]$

$u = x$, $dv = e^{-2x}dx$
$du = dx$, $v = -\frac{e^{-2x}}{2}$

$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{2}e^{-2x}$.

$\int_1^R e^{-2x}\,dx$

$\frac{3}{2}e^{-2}$ //

$= -\frac{1}{2}R^2e^{-2R} - \frac{1}{2}Re^{-2R} - \frac{1}{2}e^{-2R} + \frac{1}{2}e^{-2} + \frac{1}{2}e^{-2} + \frac{1}{2}e^{-2}$

Now $\lim_{R\to\infty} R^2e^{-2R} = \lim_{R\to\infty} \frac{R^2}{e^{2R}} \underset{\text{L'Hospital}}{=} \lim_{R\to\infty} \frac{2R}{2e^{2R}}$

$\overset{L}{=} \lim_{R\to\infty} \frac{2}{4e^{2R}} = 0$

Similarly $\lim_{R\to\infty} Re^{-2R} = 0$, and of course $\lim_{R\to\infty} e^{-2R} = 0$

$\therefore \int_1^\infty x^2e^{-2x}\,dx = \frac{3}{2}e^{-2} < \infty$ //

(26) Note that $\sec x$ is undefined at $\pi/2$.

$$\int_0^{\pi} \sec^2 x \, dx$$

$$= \int_0^{\pi/2} \sec^2 x \, dx + \int_{\pi/2} \sec^2 x \, dx$$

$$= \lim_{R_1 \to} \int \sec^2 x dx + \lim_{R_2 \to \pi/2^+} \int_{R_2} \sec^2 x \, dx$$

$$= \lim_{R_1 \to} \tan R_1 \,.\, \lim_{R_2 \to \pi/2^+} \tan R_2 .$$

$$= +\infty - (-\infty) = +\infty . // \quad \text{diverges}$$

$$\tan x = \frac{\sin x}{\cos x}$$

as $R_1 \to \pi/2^-$, $\cos x$ is very close 0, but $+$

$R_2 \to \pi/2^+$, " " " " " , " $-$.

(34) $\displaystyle\int_0^R \cos x \, e^{-\sin x} dx$

$$= \int_0^{\sin R} \cancel{\cos x} \, e^{-u} \frac{du}{\cancel{\cos x}} \qquad \text{let } u = \sin x, \; dx = du/\cos x$$

$$= -e^{-u} \Big|_0^{\sin R} = 1 - e^{-\sin R}.$$

$\displaystyle\lim_{R \to \infty} e^{-\sin R}$ doesn't exist ...

since if we let $R \to \infty$ via.

$\pi, 2\pi, 3\pi, \ldots, n\pi, \ldots$, then $\sin(n\pi) = 0$

$\Rightarrow 1 - e^{\sin R} = 1 - 1 = 0$;

however, if we let $R \to \infty$ via

$\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \ldots, \frac{(4k+1)\pi}{2}, \ldots,$

$\sin \frac{(4k+1)\pi}{2} = 1$

$\Rightarrow 1 - e^{\sin R} = 1 - e$, .

$\therefore$ two ways of letting $R \to \infty$ yield different limits

$\Rightarrow$ limit DNE.