H.W. 3 Solutions (Optional)

§6- 6, 17, 30

Routine Partial Fraction Applications.

§6-6 : 3, 13, 15, 28, 33

(3) not improper : $x^{2/5}$ is defined for all x

(13) Recall $\int \ln x \, dx = x \ln x - x$

$$\int_0^1 \ln x \, dx = \lim_{R \to 0^+} \int_R^1 \ln x \, dx$$
$$= \lim_{R \to 0^+} (-1 - R \ln R - R)$$
$$= -1 - \lim_{R \to 0^+} R \ln R$$
$$= -1 - \lim_{R \to 0^+} \frac{\ln R}{(1/R)} \quad \text{"}\frac{\infty}{\infty}\text{"}$$
$$= -1 - \lim_{R \to 0^+} \frac{1/R}{(-1/R^2)}$$
$$= -1 + \lim_{R \to 0^+} R = -1 < \infty$$

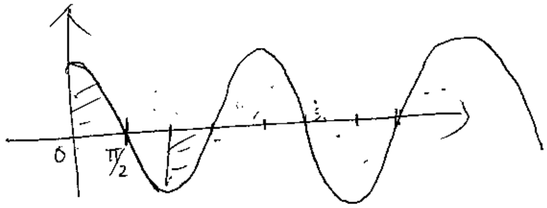
(28) $$\int_0^\infty \frac{1}{(x-2)^2} dx = \int_0^2 \frac{1}{(x-2)^2} dx + \int_2^\infty \frac{1}{(x-2)^2} dx$$

Note $$\int_0^2 \frac{1}{(x-2)^2} dx = \lim_{R \to 2^-} \int_0^R \frac{1}{(x-2)^2} dx$$
$$= \lim \int_{-2}^{R-2} \frac{1}{u^2} du \qquad u = x-2, \ du = dx$$
$$= \lim_{R \to 2^-} \left(\frac{1}{2} - \frac{1}{R-2} \right) = +\infty .$$

and since $\int_2^\infty \frac{1}{(x-2)^2} dx \geq 0$ (WHY?) $\Rightarrow \int_0^\infty \frac{1}{(x-2)^2} dx = +\infty + \text{something}$

$= +\infty$

(33) $\int_0^\infty \cos x \, dx = \lim_{R\to\infty} \int_0^R \cos x \, dx$



~~if we let $R \to \infty$ via $0, 3\pi/2, 5\pi/2, 7\pi/2, \ldots (2n+1)\pi/2, \ldots$~~
~~we see $\int_0^R \cos x \, dx = 0$ always~~

if we let $R \to \infty$ via $0, \pi/2, 2\pi/2, 3\pi/2, \ldots n\pi/2, \ldots$

$$\int_0^{n\pi/2} \cos x \, dx = \begin{cases} 0, & n \text{ even} \\ > 0, & n = 1, 5, 9, 13 \ldots \\ < 0 & n = 3, 7, 11, \ldots \end{cases}$$

$(= -\ )$

∴ limit doesn't exist. < it jumps back and forth between these 3 different numbers >