Homework 5 Solutions (Graded)

§8.1 : 30, 33, 36, 40, 41, 55.

(30) $-1 \le \cos(n\pi) \le 1$

$\therefore \quad \frac{-1}{n^2} \le \frac{\cos(n\pi)}{n^2} \le \frac{1}{n^2}$

and $\frac{1}{n^2}$ & $\frac{-1}{n^2} \to 0$. $\therefore$ By squeeze thm.,

$\frac{\cos(n\pi)}{n^2} \to 0$ //

(33) $a_n = \frac{n+3}{n+2}$

$$a_n - a_{n+1} = \frac{n+3}{n+2} - \frac{n+4}{n+3}$$

$$= \frac{(n+3)^2 - (n+2)(n+4)}{(n+2)(n+3)}$$

$$= \frac{n^2+6n+9-n^2-6n-8}{(n+2)(n+3)} = \frac{1}{(n+2)(n+3)} \geq 0$$

$\therefore$ a decreasing sequence. //

(36) $a_n = \frac{n!}{5^n}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!/5^{n+1}}{n!/5^n} = \frac{5^n}{5^{n+1}} \frac{(n+1)!}{n!} = \frac{n+1}{5} > 1 \quad \text{when } n \geq 1$$

since $a_n \geq 0$, it means $a_{n+1} > a_n$ $\forall n$.

$\therefore$ an increasing sequence. //

(40) $a_n = \frac{6n-1}{n+3}$

$$\le \frac{6n}{n} \quad \left(\begin{array}{l} 6n-1 \le 6n \\ \text{and } n+3 > n \end{array} \right)$$

$= 6$

and clearly, $0 \le a_n$.

$\therefore \quad 0 \le a_n \le 6$. bounded.

(41) $a_n = \dfrac{\sin(n^2)}{n+1}$

since $-1 \le \sin(n^2) \le 1$

$\therefore \dfrac{-1}{n+1} \le \dfrac{\sin(n^2)}{n+1} \le \dfrac{1}{n+1}$

for all $n \geq 1$, $\dfrac{1}{n+1} \le \dfrac{1}{2}$ and $\dfrac{-1}{n+1} \geq -\dfrac{1}{2}$

$\therefore \dfrac{-1}{2} \le \dfrac{\sin(n^2)}{n+1} \le \dfrac{1}{2}$

or $\left|\dfrac{\sin(n^2)}{n+1}\right| \le \dfrac{1}{2}$. //

(55) note that

$$\lim_{n\to\infty} p^n = \begin{cases} 1, & p=1 \\ 0, & -1<p<1 \\ \infty, & p>1 \\ -\infty, & p<1 \end{cases} \quad ; \ (-1)^n \text{ if } p=-1.$$

$\therefore$ For to converge, $p=1$, >1, or -1. //

§8.2: 2, 6, 10, 16.

(2) Since $\lim_{k\to\infty} \frac{1}{3}(5)^k = \infty \neq 0$,

the series diverges //

(6) $$\sum_{k=0}^{\infty} 5\left(-\frac{1}{3}\right)^k = 5 \sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k$$
$$= 5 \frac{1}{1-\left(\frac{-1}{3}\right)} = \frac{15}{4} \text{ //}$$

(10) $$\frac{9}{k(k+3)} = \frac{A}{k} + \frac{B}{k+3}$$
$$9 = A(k+3) + Bk$$
$$= (A+B)k + 3A$$
$$\Rightarrow A = 3 \ \& \ B = -3$$
$$\frac{9}{k(k+3)} = \frac{3}{k} - \frac{3}{k+3}$$
$$\sum_{k=1}^{N} \frac{3}{k} - \frac{3}{k+}$$
$$= 3\left(\frac{1}{1} - \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{5}} + \frac{1}{3} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{5}} - \frac{1}{8} \cdots + \frac{1}{N} - \frac{1}{N+3}\right)$$
$$= 3\left(1 - \frac{1}{N+3}\right)$$
letting $N \to \infty$, we see $\sum_{k=1}^{\infty} \frac{9}{k(k+3)} = 3$ //

(16) $$\sum_{k=3}^{\infty} (-1)^k \frac{3}{2^k} = 3 \sum_{k=3}^{\infty} \left(\frac{-1}{2}\right)^k$$
$$= 3 \frac{\left(\frac{-1}{2}\right)^3}{1-\left(\frac{-1}{2}\right)} = 2\left(-\frac{1}{8}\right) = -\frac{1}{4} \text{ //}$$