HW7 (Optional) Solutions

§8.4 9, 19, 39

(9) $\frac{4^k}{k^2} \to \infty$, $\quad \therefore (-1)^k \frac{4^k}{k^2} \not\to 0 \quad$ kth term test ⇒ diverges.

(19) $2e^{-k} = \frac{2}{e^k}$, which goes to 0 and decreasing

$$\left(\frac{a_{k+1}}{a_k} = \frac{1}{e} < 1 \right)$$

$\therefore$ AST ⇒ Series converges.

(39) $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ w/ $a_k = \begin{cases} \frac{1}{k} ; & k \text{ odd} \\ \frac{1}{k^2} ; & k \text{ even.} \end{cases}$

$= \sum_{k=1}^{\infty} \left[\frac{1}{2k-1} - \frac{1}{(2k)^2} \right]$ diverges,

since $\sum_{k=1}^{\infty} \frac{1}{2k-1} = \infty$ (compare to $\frac{1}{k}$)

and by Theorem 2.3 (ii). <p. 633>

$\therefore$ "decreasing" assumption is indispensible

§8.5 41, 42, 43

(41) $$\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \lim_{k\to\infty} \frac{\frac{(k+1)!}{(k+1)^{k+1}}}{\frac{k!}{k^k}}$$

$$= \lim_{k\to\infty} \frac{(k+1)!}{k!} \frac{k^k}{(k+1)^{k+1}}$$

$$= \lim_{k\to\infty} \cancel{(k+1)} \frac{1}{\cancel{k+1}} \left(\frac{k}{k+1}\right)^k = \underset{<1}{\frac{1}{e}} \quad \langle \text{use the given fact} \rangle$$

∴ Series converges absolutely.

(42) $$\lim_{k\to\infty} \frac{\frac{(k+1)!}{1\cdot 3\cdot 5\cdots(2k-1)(2k+1)}}{\frac{k!}{1\cdot 3\cdot 5\cdots(2k-1)}} = \lim_{k\to\infty} (k+1)\frac{2k-1}{2k+1}$$

$$= \infty > 1$$

∴ Series diverges.

(43) $$\lim_{k\to\infty} \frac{\frac{p^{k+1}}{k+1}}{\frac{p^k}{k}} = p$$

∴ $p<1 \Rightarrow$ series converges.

$p=1$: $\sum_{k=1}^{\infty} \frac{1}{k} = \infty$ (harmonic series)

∴ all $p<1$ make series converge.