HW8. (Optional) Solutions.

§8-5: 7, 23, 25.

(7) $\sum_{k=3}^{\infty} (-1)^k \frac{3^k}{k!}$

$$L(x) = \lim_{k\to\infty} \frac{\frac{3^{k+1}}{(k+1)!}}{\frac{3^k}{k!}} = \lim_{k\to\infty} \frac{3}{k+1} = 0 < 1$$

$\therefore$ converges absolutely $_{\prime\prime}$

(23) $\sum_{k=1}^{\infty} \frac{|\sin k|}{k^2} \le \sum_{k=1}^{\infty} \frac{1}{k^2} < \infty$ (p-series w/ $p=2$)

$\sum_{k=1}^{\infty} \frac{|\sin k|}{k^2}$ converges

$\therefore \sum_{k=1}^{\infty} \frac{\sin k}{k^2}$ converges absolutely $_{\prime\prime}$

(25) $\cos k\pi = (-1)^k$

$\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ converges conditionally.

⟨Since $\sum \frac{1}{k}$ diverges & $\sum \frac{(-1)^k}{k}$ converges⟩

§8.6: 21, 25, 29, 34, 42, 43.

(21) $\sum_{k=0}^{\infty} k!\,(x+1)^k$

$L(x) = (k+1)|x+1| = \infty$ unless $x=-1$.

$\therefore$ radius of conv. $=0$. //

(25) $L(x) = \lim_{k\to\infty} \dfrac{\frac{(k+1)!}{(2k+2)!}}{\frac{k!}{(2k)!}} |x|$

$= \lim_{k\to\infty} (k+1) \dfrac{|x|}{(2k+2)(2k+1)} = 0$

$\therefore$ converges everywhere

(29) $\sum_{k=1}^{\infty} \dfrac{4^k}{\sqrt{k}} x^k$

$L(x) = \lim_{k\to\infty} \dfrac{\frac{4^{k+1}}{\sqrt{k+1}} |x|^{k+1}}{\frac{4^k}{\sqrt{k}} |x|^k}$

$= \lim_{k\to\infty} \dfrac{4\sqrt{k}}{\sqrt{k+1}} |x| = 4|x|$.

$L(x) < 1$ if $|x| < 1/4$, ie. $-1/4 < x < 1/4$

(34) $f(x) = \dfrac{3}{(x-1)^2} = 3 \dfrac{d}{dx} \dfrac{-1}{x-1} = 3 \dfrac{d}{dx} \dfrac{1}{1-x}$

$= 3 \dfrac{d}{dx} \sum_{k=0}^{\infty} x^k$, $|x|<1$, $-1<x<1$

$= 3 \sum_{k=1}^{\infty} k x^{k-1}$ radius $= 1$

(42) $\sum_{k=0}^{\infty} a_k x^k$ converges for all $x \in (-r, r)$

$\therefore \lim_{k\to\infty} \frac{a_{k+1}}{a_k} |x| \leq 1$ for all $x \in (-r, r)$

$\quad\quad\quad\quad \| $

$|x| \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$

$\therefore |x|^2 \lim_{k\to\infty} \frac{a_{k+1}}{a_k} \leq 1$ for all $x \in (-\sqrt{r}, \sqrt{r})$

but it's exactly the "$L(x)$" for $\sum_{k=0}^{\infty} a_k x^{2k}$.

(43) $\sum_{k=0}^{\infty} a_k x^k$ converges for all $x \in (-r, r)$

$|x| \cdot \lim_{k\to\infty} \frac{a_{k+1}}{a_k} \leq 1$. " " " "

For $\sum_{k=0}^{\infty} a_k (x-c)^k$

$L(x) = |x-c| \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$

≤ 1 exactly when $|x-c| \in (-r, r)$

$\Updownarrow$

$x \in (c-r, c+r)$

$\therefore$ radius is still r.

Note: endpoints are irrelevent for radius of convergence