HW 9 (Optional) Solutions.

§8.7: 3, 31, 37

(3) $f(x) = \ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}(x+1-1)^k$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k$$

converges when $1+x \in (0,2]$ <See p. 681>

which exactly when $x \in (-1, 1]$ //

(31) $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k (1)^{2k+1}}{2k+1} = \tan^{-1}(1) = \frac{\pi}{4}$ //

(37) $\sin(2x) = \sum_{k=0}^{\infty} \frac{(-1)^k (2x)^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^k 2^{2k+1}}{(2k+1)!} x^{2k+1}$

$$\therefore \quad x\sin(2x) = \sum_{k=0}^{\infty} \frac{(-1)^k 2^{2k+1}}{(2k+1)!} x^{2k+2}$$

converges everywhere, since $\sin x$ does.