Math 241 Discussion Section-Work sheet

10/7/2008

PartI (Formula: Lagrange multiplier method)

Describe how to find the minimum value (or maximum value) of f subject to $g = 0$.

PartII (Applications)

1. Find the point(s) on the surface $z = xy + 1$ which is closest to the origin.

   (a) Set up this problem as a Lagrange multiplier problem by specifying f to be optimized (say to be maximized or minimized) and the constraint equation.

   $f = x^2 + y^2 + z^2$ minimize f

   $g = z - xy - 1 = 0$

   (b) Solve this problem using the Lagrange multiplier method.

   $\nabla f = \langle 2x, 2y, 2z \rangle$ $\quad \nabla g = \langle -y, -x, 1 \rangle$

Solve. $\begin{cases} 2x = -y\lambda & — ① \\ 2y = -x\lambda & — ② \\ 2z = \lambda & — ③ \\ z - xy - 1 = 0 & — ④ \end{cases}$

Case 1: $x = 0$

Note: $x = 0 \Rightarrow y = 0$ by ② ; $y = 0 \Rightarrow x = 0$ by ①] so $x = 0$ iff $y = 0$ — (✱)

④ $\Rightarrow$ $z - 0 - 1 = 0 \Rightarrow z = 1$ & $\lambda = 2$ by ③

$(0, 0, 1)$ is a sol.

Case 2: $x \neq 0 \Leftrightarrow y \neq 0$ By (✱)

By ① $\lambda \neq 0$.

so $\frac{x}{-y} = \frac{y}{-x} = z = \frac{\lambda}{2}$

$\Rightarrow x^2 = y^2 \Rightarrow x = \pm y$

when $x = y$ $z = -1$

④ $-1 - x^2 - 1 = 0 \Rightarrow x^2 = -2$ ✱

when $x = -y$ $z = 1$

$1 - x^2 - 1 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$ ✱

Hence $(0, 0, 1)$ is the only sol. & minimize f.