The question I did in class.

Ex1. You are standing at the point $(-100, -100, 430)$ on a hill whose shape is

$$z = 500 - 0.003x^2 - 0.004y^2$$

(a) In what direction should you proceed to climb the most steeply?

(b) At what angle from the horizontal will you initially be climbing?

Solution: Let $f(x,y) = 500 - 0.003x^2 - 0.004y^2$

(a) most steeply $\Rightarrow$ want value of the max. rates of change of z.

so it is $\|\nabla f\|$ in the direction of ∇f.

$$\nabla f = \langle -0.006x, -0.008y \rangle$$

$$\nabla f(-100, -100) = \langle 0.6, 0.8 \rangle \quad - *$$

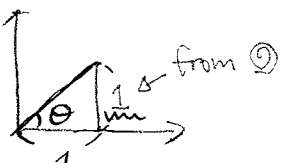
$$\|\nabla f(-100, -100)\| = \sqrt{(0.6)^2 + (0.8)^2} = \frac{1}{\text{unit}} \quad - ⓞ$$

It means that:

If you go the direction of $\langle 0.6, 0.8 \rangle$*, you will climb the most steeply with the rate $= \frac{1}{\text{unit}}$.

Ans: $\langle 0.6, 0.8 \rangle$

(b)



$$\text{rate} = \tan\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

so you are climbing in the angle of $\frac{\pi}{4}$ from the horizontal.

Ans: $\frac{\pi}{4}$

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The question I mentioned a little bit in class.

EX2: You are standing at a point $(x,y)=(100,100)$ on the side of a mountain of shape

$$z = \frac{1}{1000}(3x^2 - 5xy + y^2)$$

with x-axis pointing east and y-axis pointing north.

(a) If you head north-east will you be ascending or descending? At what angle?

(b) If you head 30° north of east will you be ascending or descending? At what angle?

Solution: Let $f(x,y) = \frac{1}{1000}(3x^2 - 5xy + y^2)$

(a) heading the direction $\vec{u}$, you are $\begin{cases} \text{ascending} & \text{if } D_{\vec{u}}f > 0 \\ \text{descending} & \text{if } D_{\vec{u}}f < 0 \end{cases}$

north-east $\rightsquigarrow$ direction $\vec{u} = \langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

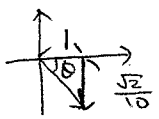
(Note: $\vec{u}$ must be a unit vector.)

$$\nabla f = \frac{1}{1000}\langle 6x-5y, -5x+2y \rangle$$

$$\nabla f(100,100) = \frac{1}{1000}\langle 600-500, -500+200 \rangle = \langle \frac{1}{10}, \frac{-3}{10} \rangle$$

$$D_{\vec{u}}f(100,100) = \nabla f(100,100) \cdot \vec{u} = \langle \frac{1}{10}, \frac{-3}{10} \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$$

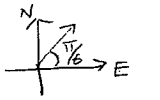
$$= \frac{1}{10\sqrt{2}} + \frac{-3}{10\sqrt{2}} = \frac{-\sqrt{2}}{10} \quad (<0 \text{ so descending})$$



$$\tan\theta = \frac{\sqrt{2}}{10} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{2}}{10}\right)$$

Ans: You are descending at the angle of $\tan^{-1}\left(\frac{\sqrt{2}}{10}\right)$

(b) direction $\vec{u}_2 = \langle \cos\frac{\pi}{6}, \sin\frac{\pi}{6} \rangle$



$$D_{\vec{u}_2}f(100,100) = \langle \frac{1}{10}, \frac{-3}{10} \rangle \cdot \langle \cos\frac{\pi}{6}, \sin\frac{\pi}{6} \rangle = \frac{1-3\sqrt{3}}{20} < 0 \text{ descending.}$$

$$\theta = \tan^{-1}\left(\frac{3\sqrt{3}-1}{20}\right)$$