Part II

Given a temperature function $T(x, y, z) = (1 - z)e^{-(x^2+y^2)}$.

1. Find the gradient of T at $(2, 0, 1)$.

$$\nabla T = \langle (1-z)\, e^{-(x^2+y^2)}(-2x)\ ,\ -2y(1-z)e^{-(x^2+y^2)}\ ,\ -e^{-(x^2+y^2)} \rangle$$

$$\nabla T\,(2,0,1) = \langle 0\ ,\ 0\ ,\ -e^{-4} \rangle$$

2. Find the directional derivative of T at $(2, 0, 1)$ in the direction from $(1, 0, 2)$ to $(3, 1, -1)$. $\vec{v} = \langle 3-1, 1-0, -1-2 \rangle = \langle 2, 1, -3 \rangle$

$$\vec{u} = \frac{1}{\sqrt{4+1+9}} \langle 2, 1, -3 \rangle$$

$$D_{\vec{u}} T\,(2,0,1) = \langle 0, 0, -e^{-4} \rangle \cdot \frac{\langle 2, 1, -3 \rangle}{\sqrt{14}} = \frac{3e^{-4}}{\sqrt{14}}$$

3. Find the directions of maximun and minimum change of T at $(2, 0, 1)$, and the values of the maximum and minimum rates of change.

$\|\nabla T(2,0,1)\| = e^{-4}$ max e^{-4} dir. $= \langle 0, 0, -1 \rangle$

min $-e^{-4}$ dir $= \langle 0, 0, 1 \rangle$

4. Find the equation of the tangent plane and the normal line to the surface of $T(x, y, z) = 0$ at $(2, 0, 1)$.

Normal vector $= \nabla T\,(2,0,1) = \langle 0, 0, -e^{-4} \rangle$

tangent plane : $-e^{-4}(z-1) = 0 \Rightarrow z = 1$

Normal line $\begin{cases} x = 2 \\ y = 0 \\ z = -e^{-4}t + 1 \end{cases}$