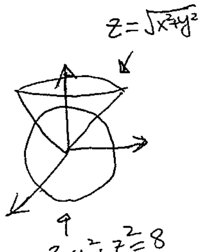
Part II

$\vec{F} = \langle x + e^{yz}, \sin xz, y \sin xy \rangle$

$\vec{\nabla} \cdot \vec{F} = 1$

#1 Q is bdd by $z = \sqrt{x^2+y^2}$ and $z = \sqrt{8 - x^2 - y^2}$



$$\iint_{\partial Q} \vec{F} \cdot \vec{n}\, ds = \iiint_Q \vec{\nabla} \cdot \vec{F}\, dV = \iiint_Q 1\, dV$$

$$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{\sqrt{8}} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= 2\pi \left(-\cos\phi \Big|_{\phi=0}^{\pi/4}\right) \left(\frac{\rho^3}{3}\right)\Big|_{\rho=0}^{\sqrt{8}} = \boxed{\frac{32\pi}{3}(\sqrt{2}-1)}$$

#2. Q is bdd by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$

$$\iint_{\partial Q} \vec{F} \cdot \vec{n}\, ds = \iiint_Q \vec{\nabla} \cdot \vec{F}\, dV = \iiint_Q 1\, dV$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{2} \int_{z=r^2}^{8-r^2} 1 \cdot r\, dz\, dr\, d\theta$$

$x^2+y^2 = 8 - x^2 - y^2$

$\Rightarrow x^2+y^2 = 4$

so $0 \le r \le 2$

$$= 2\pi \int_{r=0}^{2} (8 - 2r^2) r\, dr = 2\pi \left(4r^2 - \frac{r^4}{2}\right)\Big|_{r=0}^{2}$$

$$= \boxed{16\pi}$$

If $\vec{F} = \langle x^2 + e^{yz}, \sin xz, y \sin xy \rangle$, do the same Q for #1, #2

$\vec{\nabla} \cdot \vec{F} = 2x$

#1 $\iint_{\partial Q} \vec{F} \cdot \vec{n}\, ds = 0$

#2 $\iint_{\partial Q} \vec{F} \cdot \vec{n}\, ds = 0$

Remark:

Knowing $\int_{x=c}^{d} \int_{y=a}^{b} f(x) g(y)\, dy\, dx = \int_{x=c}^{d} f(x)\, d \int_{y=a}^{b} g(y)\, dy$ will simplify the computations.

But this is only true if the range / limit are independent from each other and the function can be written as product $f(x) g(y)$