Part I

#2. From #1

we know $\vec{n} = \left\langle \frac{x}{\sqrt{a^2-x^2-y^2}}, \frac{y}{\sqrt{a^2-x^2-y^2}}, 1 \right\rangle \times \frac{1}{\left\| \left\langle \frac{x}{\sqrt{a^2-x^2-y^2}}, \frac{y}{\sqrt{a^2-x^2-y^2}}, 1 \right\rangle \right\|}$

use $x = a\sin\phi\cos\theta$, $y = a\sin\phi\sin\theta$, $z = a\cos\phi$

$$\Rightarrow \vec{n} = \frac{\left\langle \frac{\sin\phi\cos\theta}{\cos\phi}, \frac{\sin\phi\sin\theta}{\cos\phi}, 1 \right\rangle}{\left\| \left\langle \frac{\sin\phi\cos\theta}{\cos\phi}, \frac{\sin\phi\sin\theta}{\cos\phi}, 1 \right\rangle \right\|} = \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi \rangle$$

$$\iint_{S_1} \vec{F}\cdot\vec{n}\, dS$$

$$= \int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/2} \underbrace{\langle a\sin\phi\cos\theta, a\sin\phi\sin\theta, a\cos\phi \rangle}_{\vec{F}} \cdot \underbrace{\langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi \rangle}_{\vec{n}} \underbrace{a^2\sin\phi\, d\phi\, d\theta}_{dS}$$

$$= \int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/2} a\cdot a^2\sin\phi\, d\phi\, d\theta = 2\pi a^3$$

Remark:

You also can assume $\vec{n} = \pm \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi \rangle$ (± depends on the orientation)

is a unit normal vector of a sphere centered at the origin (with any radius).

without doing computation.

Likewise, $\langle x, y, z \rangle$ is always a normal vector of a sphere centered at $(0,0,0)$