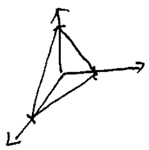
Part I

11/20/08

#1. mass $m = \iint_S \delta \, dS = \iint_S y \, dS = \int_{x=-1}^{1} \int_{y=x^2}^{1} y \cdot \| \langle -1,-2,-1 \rangle \| \, dy \, dx$

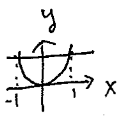
$= \frac{4}{5}\sqrt{6}$



$S: x+2y+z=4$

$z = 4-x-2y$

$\langle f_x, f_y, -1 \rangle = \langle -1,-2,-1 \rangle$



$\bar{x} = \frac{1}{m} \iint_S x \delta \, dS = \frac{5}{4\sqrt{6}} \int_{x=-1}^{1} \int_{y=x^2}^{1} xy \cdot \| \langle -1,-2,-1 \rangle \| \, dy \, dx = 0$

$\bar{y} = \frac{1}{m} \iint_S y \delta \, dS = \frac{5}{7}$

$\bar{z} = \frac{1}{m} \iint_S z \delta \, ds = \frac{1}{m} \iint_S zy \, ds = \frac{5}{4\sqrt{6}} \int_{x=-1}^{1} \int_{y=x^2}^{1} (4-x-2y) y \cdot \| \langle -1,-2,-1 \rangle \| \, dy \, dx = \frac{18}{7}$

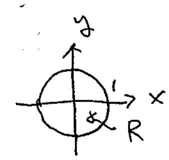
#2. $S: x^2+y^2+z^2 = 9 \qquad dS = 3^2 \sin\phi \, d\phi \, d\theta = 9 \sin\phi \, d\phi \, d\theta$

$\iint_S \sqrt{x^2+y^2+z^2} \, ds = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\pi/2} \sqrt{9} \cdot 9 \sin\phi \, d\phi \, d\theta = \frac{27\pi}{2}$

Part II

$S: z = 1-x^2-y^2$

a normal vector $\pm \langle f_x, f_y, -1 \rangle = \pm \langle -2x, -2y, -1 \rangle$



① $\vec{n}$ is downward pointing $\Rightarrow \vec{n} = \frac{\langle -2x, -2y, -1 \rangle}{\| \langle -2x, -2y, -1 \rangle \|}$

② $ds = \| \langle -2x, -2y, -1 \rangle \| \, dA$

③ $\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_R \langle x(1-x^2-y^2), (1-x^2-y^2)y, 2(1-x^2y^2)^2 \rangle \cdot \frac{\langle -2x, -2y, -1 \rangle}{\cancel{\| \langle -2x, -2y, -1 \rangle \|}} \cancel{\| \langle -2x, -2y, -1 \rangle \|} \, dA$

$= \iint_R -2(1-x^2-y^2) \, dA = \int_{\theta=0}^{2\pi} \int_{r=0}^{1} -2(1-r^2) \, r \, dr \, d\theta = -\pi$