

Part III | Two ways to correct the typo.

(A.) Compute ~~I_y~~ \rightsquigarrow I_x

$$I_x = \int_C \omega^2 \rho \, ds = \int_C y^2 \cdot y \, ds$$

step 1: parametrize C

$$\begin{cases} x(t) = t \\ y(t) = 4 - t^2 \end{cases} \quad 0 \leq t \leq 2$$

step 2: $ds = \sqrt{(x'(t))^2 + (y'(t))^2} \, dt = \sqrt{1^2 + (-2t)^2} \, dt = \sqrt{1+4t^2} \, dt$

step 3:

$$I_x = \int_C y^3 \, ds = \boxed{\int_{t=0}^2 (4-t^2)^3 \sqrt{1+4t^2} \, dt}$$

(B) w = distance from (x, y) to ~~x -axis~~ \rightsquigarrow y -axis.

$$I_y = \int_C \omega^2 \rho \, ds = \int_C x^2 \cdot y \, ds$$

Note that, step 1 & step 2 are the same.

Step 3:

$$I_y = \int_C x^2 y \, ds = \boxed{\int_{t=0}^2 t^2 (4-t^2) \sqrt{1+4t^2} \, dt}$$