#29. $\vec{r}(t) = \langle 2\cos t, 3\sin t \rangle$ Find max./min. curvature.

$\vec{r}' = \langle -2\sin t, 3\cos t \rangle$ $\quad \|\vec{r}'\| = \sqrt{4\sin^2 t + 9\cos^2 t} = \sqrt{4 + 5\cos^2 t}$

$$\vec{T} = \frac{\vec{r}'}{\|\vec{r}'\|} = \left\langle \frac{-2\sin t}{\sqrt{4+5\cos^2 t}}, \frac{3\cos t}{\sqrt{4+5\cos^2 t}} \right\rangle$$

$$\vec{T}': \frac{d}{dt}\left(\frac{-2\sin t}{\sqrt{4+5\cos^2 t}}\right) = \frac{-2\cos t\left(\sqrt{4+5\cos^2 t}\right) - (-2\sin t)\frac{1}{2}(4+5\cos^2 t)^{-\frac{1}{2}}(10\cos t(-\sin t))}{4+5\cos^2 t} \times \frac{1}{\sqrt{4+5\cos^2 t}}$$

$$= \frac{1}{(4+5\cos^2 t)^{3/2}}\left(-8\cos t \underbrace{-10\cos^3 t - 10\sin^2 t\cos t}_{-10\cos t}\right)$$

$$= \frac{-18\cos t}{(4+5\cos^2 t)^{3/2}}$$

$$\frac{d}{dt}\left(\frac{3\cos t}{\sqrt{4+5\cos^2 t}}\right) = \frac{-12\sin t - 15\sin t\cos^2 t + 15\cos^2 t\sin t}{(4+5\cos^2 t)^{3/2}} = \frac{-12\sin t}{(4+5\cos^2 t)^{3/2}}$$

$$\|\vec{T}'\| = \sqrt{\frac{18^2\cos^2 t}{(4+5\cos^2 t)^3} + \frac{12^2\sin^2 t}{(4+5\cos^2 t)^3}} = \sqrt{\frac{(18^2-12^2)\cos^2 t + 12^2}{(4+5\cos^2 t)^3}}$$

$$= \sqrt{\frac{2^2(9^2-6^2)\cos^2 t + 2^2 6^2}{(4+5\cos^2 t)^3}} = \frac{2\sqrt{(81-36)\cos^2 t + 36}}{(4+5\cos^2 t)\sqrt{4+5\cos^2 t}}$$

$$= \frac{2\sqrt{45\cos^2 t + 36}}{(4+5\cos^2 t)\sqrt{4+5\cos^2 t}} = \frac{2\sqrt{9(5\cos^2 t + 4)}}{(4+5\cos^2 t)\sqrt{4+5\cos^2 t}} = \frac{2\sqrt{9}}{4+5\cos^2 t} = \frac{6}{4+5\cos^2 t}$$

$$\kappa = \frac{\|T'\|}{\|\vec{r}'\|} = \frac{6}{4+5\cos^2 t} \times \frac{1}{\sqrt{4+5\cos^2 t}} = \frac{6}{(4+5\cos^2 t)^{3/2}}$$

max when $(4+5\cos^2 t)^{3/2}$ is min

min when $(4+5\cos^2 t)^{3/2}$ is max

$(4+5\cos^2 t)^{3/2}$ min when $\cos^2 t = 0$ then $(4+0)^{3/2} = 8$ $\quad \kappa = \frac{6}{8} = \frac{3}{4}$

$0 \le \cos^2 t \le 1$ since $-1 \le \cos t \le 1$ $\quad t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $\quad$ at $(0, \pm 3)$

max when $\cos^2 t = 1$ then $(9)^{3/2} = 27$ $\quad \kappa = \frac{6}{27} = \frac{2}{9}$

$t = 0, \pi, 2\pi, \ldots$ $\quad$ at $(\pm 2, 0)$