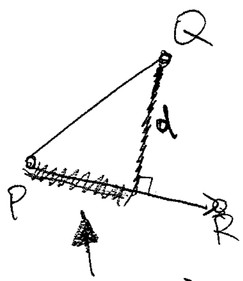
- distance from a point to a line.

(in §10.5 have .. "distance between two lines)

Distance from Q to the line through P and R.

$= d = \|\vec{PQ}\| \sin\theta.$



$\text{comp}_{\vec{PR}} \vec{PQ}$

$$\|\vec{PQ} \times \vec{PR}\| = \|\vec{PQ}\|\ \|\vec{PR}\| \sin\theta$$

$$= \|\vec{PR}\| \cdot d$$

$$\Rightarrow \boxed{d = \frac{\|\vec{PQ} \times \vec{PR}\|}{\|\vec{PR}\|}}$$

Recall: $\text{com}_{\vec{PR}} \vec{PQ} = \dfrac{\vec{PR} \cdot \vec{PQ}}{\|\vec{PR}\|}$

#22: $Q = (2,0,1)$ line through $(1,-2,2)$ & $(3,0,2)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\overset{\|}{P}\qquad\qquad\overset{\|}{R}$

$\vec{PQ} = \langle 2-1, 0-(-2), 1-2 \rangle = \langle 1, 2, -1 \rangle$

$\vec{PR} = \langle 3-1, 0-(-2), 2-2 \rangle = \langle 2, 2, 0 \rangle$

$\|\vec{PR}\| = \sqrt{2^2+2^2+0^2} = \sqrt{4+4} = \sqrt{8}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 2 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix}$$

$$= +2\vec{i} - (2)\vec{j} + \vec{k}(2-4)$$

$\|\vec{PQ} \times \vec{PR}\| = \sqrt{2^2 + (-2)^2 + (-2)^2} = \sqrt{12}$

$$d = \frac{\|\vec{PQ} \times \vec{PR}\|}{\|\vec{PR}\|} = \frac{\sqrt{12}}{\sqrt{8}} = \sqrt{\frac{\cancel{12}\,3}{\cancel{8}\,2}} = \sqrt{\frac{3}{2}}. \quad \square$$