Notes on Relation of lines.

Given any two lines ℓ_1, ℓ_2 which is parallel to $\vec{a}_1$, $\vec{a}_2$ (resp.)

We say: ① ℓ_1 and ℓ_2 are <u>parallel</u> $\Leftrightarrow$ $\vec{a}_1$ and $\vec{a}_2$ are parallel

② ℓ_1 and ℓ_2 <u>intersect</u> if there is a intersection

↳ In this case, we may ask (where is the intersection?
angle between ℓ_1, ℓ_2? orthogonal?

③ ℓ_1 and ℓ_2 are <u>skew</u> if neither ① nor ②
(i.e. nonparallel, nonintersecting)

<u>Example:</u>

Determine whether $\ell_1: \begin{cases} x = 1+t \\ y = 2-t \\ z = 1+3t \end{cases}$ and $\ell_2: \begin{cases} x = 1+4s \\ y = 2-5s \\ z = -1+5s \end{cases}$ parallel, intersect, or skew.

<u>RK</u>: ℓ_1 & ℓ_2 are two different lines, you should use two different parameters (e.g. t & s)

<sol>:

$\vec{a}_1 = \langle 1, -1, 3 \rangle$ $\quad \vec{a}_2 = \langle 4, -5, 5 \rangle$

$4\vec{a}_1 = \langle 4, -4, 12 \rangle \neq c\vec{a}_2$ for any c $\Rightarrow$ $\vec{a}_2$ & $\vec{a}_1$ not parallel — (*)

Intersect?

If ℓ_1, ℓ_2 intersect, then there exists some s & t such that $\begin{cases} x = 1+t = 1+4s & -① \\ y = 2-t = 2-5s & -② \\ z = 1+3t = -1+5s & -③ \end{cases}$ holds.

① $\Rightarrow$ $t = 4s$

plug into ② $\Rightarrow$ $2-(4s) = 2-5s \Rightarrow s = 0$ & $t = 4s = 0$

so ③ $\Rightarrow$ $1+0 = -1+0$ $\rightarrow\!\!\!\!\times$ (impossible!)

So no such t, s exist. $\Rightarrow$ not intersection.

Hence ℓ_1, ℓ_2 do not intersect — (**)

By def & (*) & (**), ℓ_1 and ℓ_2 are skew.

□