2. Determine whether or not the vector field is conservative. If it is, find a potential function. (problem 23-24)

(a) $\vec{F} = \frac{\vec{r}}{r^3}$. $= \left\langle \frac{x}{(x^2+y^2)^{3/2}}, \frac{y}{(x^2+y^2)^{3/2}} \right\rangle \stackrel{?}{=} \vec{\nabla} f$

$$\begin{cases} f_x = \frac{x}{(x^2+y^2)^{3/2}} & \text{--- } ① \\ f_y = \frac{y}{(x^2+y^2)^{3/2}} & \text{--- } ② \end{cases}$$

antiderivative ① $\Rightarrow$ $f(x,y) = \int \frac{x}{(x^2+y^2)^{3/2}}\,dx = -(x^2+y^2)^{-\frac{1}{2}} + g(y)$ (✡)

so $f_y = \frac{\partial}{\partial y}\left(-(x^2+y^2)^{-\frac{1}{2}} + g(y)\right) \stackrel{②}{=} \frac{y}{(x^2+y^2)^{3/2}}$

$\Rightarrow$ $y(x^2+y^2)^{-\frac{3}{2}} + g'(y) = \frac{y}{(x^2+y^2)^{3/2}}$ $\Rightarrow g'(y) = 0 \Rightarrow g(y) = C$

So plug $g(y) = C$ into (✡)

$f(x,y) = -(x^2+y^2)^{-\frac{1}{2}} + C = -r^{-1} + C$ potential function

So $\vec{F}$ is conservative. ▨

(b) $\vec{F} = \langle z^2 + 2xy, x^2 - z, 2xz - 1\rangle$.