Bounds on least pseudo-Anosov dilatations

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Given genus $g \geq 2$, $\exists\ c_g$, *a constant depending on* g, *such that*

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