Math 231
First Group Assignment
Due Friday, January 26

(1) Two different methods of integrating a function $f(x)$ may yield antiderivatives $G(x)$ and $H(x)$ quite different in appearance. How can you reconcile these antiderivatives? What would you be able to conclude from the following methods?:
  (a) Calculating numerical values of $G(x)$ and $H(x)$ for selected values of x.
  (b) Graphing $G(x)$ and $H(x)$ simultaneously. What would it mean if the two graphs did not coincide, but intersect at one or more isolated points?
  (c) Calculating the numerical values of $\int_a^b f(x)dx$, $G(b)-G(a)$, and $H(b) - H(a)$ for selected vaules of a and b.
  (d) Differentiating G(x) and H(x).

(2) The substitution $u = x^2$, $x = \sqrt{u}$, $dx = du/(2\sqrt{u})$ appears to lead to the result

$$\int_{-1}^{1} x^2 dx = 0.$$

Do you believe this result? What went wrong? Using the substitution correctly, evaluate the integral.

(3) Given an integral $\int f(x)dx$, state in your own words your strategy for factoring the integrand into a product $f(x) = g(x)h(x)$ so that the separation into parts $u = g(x)$, $dv = h(x)dx$ is effective in evaluating the integral.

(4) Give an example of an integral $\int f(x)dx$ for which the integrand can be factored in at least three different ways– some that lead to a successful integration by parts and some that do not.

(5) Apply the reduction formula in 7.4 Problem 53 to show that for each positive integer n,

$$\int_0^{\pi/2} sin^{2n} x dx = \frac{\pi}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{2n-1}{2n}$$

and

$$\int_0^{\pi/2} sin^{2n+1} x dx = \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{6}{7} \cdots \frac{2n}{2n+1}.$$

(6) For each positive integer k, let

$$I_k = \int_0^{\pi/2} sin^k x dx.$$

(a) Show that $I_{2n} \geq I_{2n+1} \geq I_{2n+2}$ for each positive integer n.
(b) Use problem 5 to show that

$$\lim_{n \to \infty} \frac{I_{2n+2}}{I_{2n}} = 1.$$

(c) Conclude from parts (a) and (b) and the Squeeze Law for Limits that

$$\lim_{x \to \infty} \frac{I_{2n+1}}{I_{2n}} = 1.$$

(d) Conclude from part (c) and problem 5 that

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots .$$

(7) Classify the types of trigonometric integrals that can be evaluated using the methods of section 7.4. Give examples of several trig integrals that cannot be evaluated using these methods.

Group 1: #1, 4
Group 2: #2, 5
Group 3: #3, 7
Group 4: #6
Group 5: #1, 7
Group 6: #2, 4
Group 7: #3, 5
Group 8: #6