Math 231
Sixth Group Assignment
Due Friday, April 6

(1) Find the area of the cirlce $r = \sin\theta + \cos\theta$ by integration in polar coordinates. Check your answer by writing the equation of the circle in rectangular coordinates, finding its radius, and then using the familiar formula for the area of a circle.

(2) The spiral of Archimedes, shown in Fig. 9.3.19 on page 644, has the equation $r = a\theta$ (a is constant). Let A_n denote the area bounded by the nth turn of the spiral, where $2(n-1)\pi \le \theta \le 2n\pi$, and by the portion of the polar axis joining its endpoints. For each $n \ge 2$, let $R_n = A_n - A_{n-1}$ denote the area between the $(n-1)$th and the nth turns. Derive the following results of Archimedes:
   (a) $A_1 = \frac{1}{3}\pi(2\pi a)^2$
   (b) $A_2 = \frac{7}{12}\pi(4\pi a)^2$
   (c) $R_2 = 6A_1$
   (d) $R_{n+1} = nR_2$ for $n \ge 2$.

(3) Refer to figure 9.3.21 on page 644, which illustrates the first turn of the logaritmic spiral $r = 2e^{-\theta/10}$. Find the areas of the two shaded regions and verify that their sum is the area of the annular region between the two curves.

(4) A *trochoid* is traced by a point P on a spoke of a wheel of radius a as it rolls along the x-axis.
   (a) If the distance of P from the center of the rolling wheel is $b > 0$, show that the trochoid is describes by the parametric equations $x = at - b\sin t$, $y = a - b\cos t$.
   (b) What does the trochoid look like when $b = a$? Now for $b > a$, experiment with different values of a and b. What determines whether the trocoid has loops, cusps, or neither?

(5) The curve C is determined by the parametric equations $x = e^{-t}$, $y = e^{2t}$. Calculate dy/dx and d^2y/dx^2 directly from these parametric equations. Conclude that C is concave upward at every point. Then sketch C.