

# Weight Filtrations in Algebraic $K$ -Theory

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**ABSTRACT.** We survey briefly some of the  $K$ -theoretic background related to the theory of mixed motives and motivic cohomology.

## 1. Introduction

The recent search for a motivic cohomology theory for varieties, described elsewhere in this volume, has been largely guided by certain aspects of the higher algebraic  $K$ -theory developed by Quillen in 1972. It is the purpose of this article to explain the sense in which the previous statement is true, and to explain how it is thought that the motivic cohomology groups with rational coefficients arise from  $K$ -theory through the intervention of the Adams operations. We give a basic description of algebraic  $K$ -theory and explain how Quillen's idea [42] that the Atiyah-Hirzebruch spectral sequence of topology may have an algebraic analogue guides the search for motivic cohomology.

There are other useful survey articles about algebraic  $K$ -theory: [50, 46, 23, 56, 40].

I thank A. Beilinson, W. Dwyer, E. Friedlander, S. Lichtenbaum, R. McCarthy, S. Mitchell, C. Soulé, and R. Thomason for frequent and useful consultations.

## 2. Constructing topological spaces

In this section we explain the considerations from combinatorial topology that give rise to the higher algebraic  $K$ -groups. The first principle is simple enough to state but hard to implement: when given an interesting group (such as the Grothendieck group of a ring) arising from some algebraic situation, try to realize it as a low-dimensional homotopy group (especially  $\pi_0$  or  $\pi_1$ )

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1991 *Mathematics Subject Classification.* Primary 19-02; Secondary 19E20.

Supported by NSF grant DMS 90-02715.

This paper is in final form and no version of it will be submitted for publication elsewhere.

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0082-0717/94 \$1.00 + \$.25 per page

