Math 385 **Section N1** **Exam 1**

Date: 09/25/2007 NAME: _____

Instructions: Show ALL your working and make your explanations as full as possible, unless the questions says otherwise. There is a total of five problems, be sure to do all five for full credit.

Problem 1	1a	_____
	1b	_____
Problem 2	2a	_____
	2b	_____
Problem 3	3a	_____
	3b	_____
Problem 4	4a	_____
	4b	_____
Problem 5	5a	_____
	5b	_____
Total		_____

Problem 1:

A ball is fired upward with an initial velocoty v_0 from the top of a building of height h. The ball falls to the ground at the base of the building. Find

a) the maximum height of the ball above the ground and

b) the total time the ball is in the air before hitting the ground.

(Hint: The ODE describing this is given by Newton's equation, $v' = -g$, where g is the earth's gravitational acceleration and $v = y'$ is the velocity, given by the derivative of the height y of the ball.)

Problem 2:

Consider the initial value problem

$$\frac{dy}{dx} = \frac{x}{y}, \quad y(1) = 1.$$

a) Determine whether the existence and uniqueness theorem does or does not guarantee the existence of a unique solution.

b) If existence is guaranteed, solve the initial value problem.

Problem 3:

Consider the autonomous differential equation

$$y' = y - y^3$$

a) Find the equilibrium solutions and show whether they are asymptotically stable or unstable (a sketch helps here!).

b) Without (!) solving the ODE explicitly, what is your guess for large time limit of a solution of the above ODE with initial value $y(0) = 10^{-2007}$. Explain.

Problem 4:

Solve the equations

a) $y' - 2xy = xe^{x^2}$.

b) $x^2y' = xy + x^2e^{y/x}$.

Problem 5:

Suppose you fire a ball upwards from a very very high cliff and you also know that the ball is, in addition to the gravitational pull of the earth, also subject to air-resistance which we assume is proportional to its velocity, $F_r = -kv$.

a) What is the differential equation for the velocity v of the ball.

b) What is the limiting velocity of the ball as it falls down.