Math 385 **Section N1** **Exam 2**

Date: 10/25/2007 NAME: ______________________________

Instructions: Show ALL your working and make your explanations as full as possible, unless the questions says otherwise. There is a total of four problems, be sure to do all four for full credit.

Problem 1	1a	________
	1b	________
	1c	________
Problem 2	2a	________
	2b	________
	2c	________
Problem 3	3a	________
	3b	________
	3c	________
Problem 4	4a	________
	4b	________
	4c	________
Total		________

Problem 1:

a) Find the general solution of the homogenous second order ODE

$$y'' + 2y' + y = 0.$$

b) What is the solution of the above ODE with $y(0) = 1$ and $y'(0) = 0$?

c) Consider now the inhomogenous ODE

$$y'' + 2y' + y = x^2 e^{-x}.$$

What is the correct guess for the form of a particular solution y_p of this non-homogenous ODE?

(Hint: in Problem c) you don't have to calculate the particular solution, you only have to write down the form which is guaranteed to find the particular solution.)

Problem 2:

a) Find the general solution of the equation

$$y'' + \omega_0^2 y = 0.$$

b) What is the correct form for the guess of a particular solution of

$$y'' + \omega_0^2 y = F_0 \sin(\omega t)$$

with $\omega \neq \omega_0$? (Just write down the correct form, do not calculate the particular solution.)

c) Find a particular solution of the equation

$$y'' + \omega_0^2 y = F_0 \sin(\omega_0 t)$$

(with $\omega_0 \neq 0$.) Sketch the particular solution you've found. What happens with the amplitude of your particular solution and what is this effect called?

Problem 3:

a) What is the general solution of the ODE

$$y^{(5)} + 2y''' + y' = 0.$$

b) What is the correct form for the guess of a particular solution y_p of

$$y^{(5)} + 2y''' + y' = x^{42}$$

c) What is the correct form for the guess of a particular solution y_p of

$$y^{(5)} + 2y''' + y' = \sin(x)$$

Problem 4:

Let p and q be continuous functions on an open interval I and consider the ODE

$$y'' + p(x)y' + q(x)y = 0$$

on this interval.

a) Let y_1 and y_2 be two solutions of the above ODE and $W[y_1, y_2] := y_1y_2' - y_1'y_2$ their Wronskian. Show that W solves the differential equation

$$W'(x) = -p(x)W(x) \quad \text{for all } x \in I.$$

b) Solve the above ODE for W and conclude that either W is identically zero on the inteval I or never zero on I.

c) Let y_1 and y_2 be linearly independent solutions of the second order ODE $y'' + p(x)y' + q(x)y = 0$. Show that there is no point $x_0 \in I$ such that $y_1(x_0) = 0$ and $y_2(x_0) = 0$.