Differential Equations, Math 441, Homework 1

Due: Friday January 27, in the beginning of the class.

Problem 1: [Variation of constants] (12 points)
Consider the differential equation

$$y' = ay + b$$

for some contants $a, b \in \mathbb{R}$ and initial condition $y(0) = y_0$.
We know a solution for the differential equation

$$y_1' = ay_1,$$

namely $y_1(t) = e^{at}$. Make the ansatz $y(t) = v(t)y_1(t)$.

i) Find a differential equation for v

ii) Solve the differential equation for v.

iii) Use this to find the solution for the differential equation $y' = ay + b$.

Problem 2: (10 points) Let $I = (\alpha, \beta)$ be an open interval and $p : I \to \mathbb{R}$ a continuous function. Show that for any choice of $t_0 \in I$, any solution of the equation

$$\mu' = p\mu \quad \text{on } I$$

is a constant multiple of the function $\exp(\int_{t_0}^{t} p(s)\, ds)$.

The following problems are from Boyce and DiPrima, 8th edition. Hand in only the problems marked with an asterix (*). Each problem is worth 5 points.

Sec. 1.1: Problems 7*, 17*, 19*, 20.

Sec. 1.2: Problem 19*.

Sec. 1.3: Problems 1*, 3*, 5, 19.