Differential Equations, Math 441, Homework 4

Due: Monday February 20, in the beginning of the class.

Each problem is worth 5 points, except problem 1, problem 2, and Section 2.5 Problem 23 A total of 50 points.
For full credit you have to show how you came up with your solution, just giving the right answer is not enough, even in real life!

Problem 1: Consider the first order ODE $y' = f(t, y)$, where f is defined on some rectangle containing the initial point (t_0, y_0) and obeys the conditions of Picard's theorem. Let y be a solution of the above ODE with initial condition $y(t_0) = y_0$ and z a solution with initial condition $z(t_0) = z_0$ (Here z_0 is so close to y_0 that $(t_0, z_0) \in R$). By Picard's theorem, we know that the two solutions exist, at least for some small time interval around t_0. Show that y and z are not too far from each other, i.e., derive a bound on $|y(t) - z(t)|$ in terms of $|y_0 - z_0|$. (Hint: Argue similar as we did in class when we proved that $y(t) = z(t)$ if $y_0 = z_)$.)

(15 points)

Problem 2: Consider the ODE $y' = \frac{y}{2} + t$, with initial condition $y(0) = 0$.
a) Compute the first three, say, Picard iterates. Based on your computation, make a guess for the n^{th} Picard iterate AND prove your guess with the help of induction. (9 points)

b) Express the limit of the Picard iterates from part a) in terms of elementary functions. Check that it is indeed a solution of the ODE. (6 points)

The following problems are from Boyce and DiPrima, 8^{th} edition. Hand in only the problems marked with an asterix (*).

Sec. 2.5: Problems 21, 23* (worth 10 points), 28.

Sec. 2.6: Problems 18*, 22*