TEACHING STATEMENT

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The Notices of the American Mathematical Society recently published a lecture given by Freeman Dyson on the different species of mathematicians. There he defines two main categories of mathematicians: the birds and the frogs. Birds are the theory builders, they fly high above the ground and see connections between different areas and concepts. Frogs are the problem solvers, mathematicians that dig deep into the ground and try to solve specific hard problems. It is impossible to tell whether one kind is better or more useful than the other simply because mathematics needs both a well-built supporting theory that unifies concepts as well as a deep understanding of specific problems.

Even though the research tastes of different mathematicians might vary, I believe that when it comes to learning mathematics more universal principles apply: one has to learn both how to fly and how to dig into the ground. Hard problems cannot be solved without thorough understanding of the underlying theory and, conversely, applying an abstract and general theory to specific problems helps the learner get a much better grasp of it. In my personal teaching experience I always tried to remain faithful to this principle. Even though I taught recitation sessions of calculus which are heavily oriented towards problem solving, I would always take some time to make sure that my students knew the basic concepts before applying them to explicit examples.

Another observation that I have made both as a learner and as a teacher is that textbooks can sometimes be a bit hard to penetrate. This is especially true for old fields of mathematics where continuous efforts over the years have optimized the arguments making them short and elegant (what Erdős would call a *Book proof*). Even though this is in principle something very positive, when it comes to learning it might give the reader the impression that proving a mathematical result requires magical abilities. A very important task of a math teacher is to guide the students through theorems and problems by highlighting the key points and ideas behind their proofs and solutions before supplying all the details. When I was teaching Calculus II, I was asked by the professor who was in charge of the course to cover the theorem that states that every bounded and monotonic sequence of real numbers converges. Before giving any formal proof, I announced to the class that I was an increasing sequence and starting walking towards the wall - my supremum - which I eventually had to reach - at least in the infinitesimal sense. I believe that when it comes to understanding a mathematical statement, heuristic arguments are just as important as rigor. After all, in real life nobody does textbook-style mathematics.

I was once told by a French language teacher that she deliberately speaks only in French to her students, even to beginners that are completely unfamiliar with the language. I was struck to hear that this method is actually effective, but then I realized that this is how everybody learns their mother tongue. However, I

believe that this technique can be applied only to highly motivated students, in an advanced undergraduate or a graduate class for example. It is unfortunate but true that students not majoring in math sometimes have a negative and stereotypical perception of mathematics as being some sort of cryptic language and view the math courses they have to take as a necessary evil. I see it as my duty to convince them otherwise and share with them my excitement for the subject. For this reason I find it very useful to introduce mathematical ideas via concepts that students are familiar with. For example, before formally discussing infinite series in one of my Calculus classes I stated Zeno's dichotomy paradox and then explained to the class the mathematics behind it.

As a student I always found it very stimulating when my teacher talked to the class about concepts a bit outside the syllabus. In my junior high school I remember my math teacher casually mentioning complex numbers and continued fractions. Later, during my college and graduate school years I recall some of my teachers briefly discussing open problems and more advanced theory. Giving a glimpse of the entire world that lies just a bit further is a tool that, if employed skillfully, can greatly increase the students motivation and appetite for further learning. In a similar direction, I always enjoy pushing my students to think about some hard and conceptual problems. I believe that teaching is not just about passing on some concise piece of knowledge, but also about intriguing students' minds, urging them to become individual thinkers and sharpening their critical skills.

Teaching is a journey that I have greatly enjoyed so far. I am grateful for all my teaching experiences and I find them very rewarding. As a teacher I have the unique opportunity to stimulate my students' intellectual curiosity and help them guide themselves through the exciting world of mathematics. But also, it is surprising and refreshing how much better I have to understand ideas before being able to teach them. For these reasons I look forward to my new teaching adventures.