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**Rigid analytic uniformization of curves and the study of isogenies. (English summary)**

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This is a beautifully written paper introducing the reader to the uniformization of abelian varieties—in particular, Jacobians of Mumford curves—and their isogenies in the context of rigid analytic geometry. The paper provides a short introduction into rigid analytic geometry over a complete normed field $k$, justifying the concepts through analogy with the complex case. The author then discusses an analytic torus, given as the quotient of the analytic group variety $(k^*)^n$ modulo a lattice. He shows that isogenies among analytic tori admit a concrete description in terms of their defining lattices. The computational advantage of this description is that it does not depend on the entries defining the lattices, but only on their norms. Then follows a section on the Jacobian of a Mumford curve $C$ of genus $g$, the discussion of which is again preceded by a comparison with the classical case, due to Abel-Jacobi, for curves over $\mathbb{C}$. A Mumford curve is a curve with totally split reduction, given as the quotient of the $\Gamma$-ordinary points on $\mathbb{P}^1_k$ modulo a Schottky group $\Gamma \subseteq \mathrm{PGL}_2(k)$. The new ingredient required for the rigid case is the theta function

$$\Theta(a, b; z) := \prod_{\gamma \in \Gamma} \frac{z - \gamma(a)}{z - \gamma(b)}$$

with the aid of which one can construct any automorphic form on the ordinary points of $\mathbb{P}^1_k$. The author shows that the dual Schottky group $\check{\Gamma}$ is isomorphic with $(k^*)^g$ and there is a surjective map $\check{\Gamma} \to \mathrm{Pic}_0(C)$ onto the Picard group of degree zero divisor classes on $C$, given by sending $c \in \check{\Gamma}$ to the unique divisor class of an automorphic form with factor $c$. This gives $\mathrm{Pic}_0(C)$ the structure of an analytic torus. Since this analytic torus is in fact algebraic by the rigid analytic version of GAGA, it is the Jacobian of $C$. There is also an explicit birational map from this Jacobian to the $g$-th symmetric product $C^{(g)}$ of the Mumford curve. The paper concludes with a discussion of the pro and contras of rigid analytic uniformization, accompanied by some concrete computations for hyperelliptic curves.

Reviewed by *Hans Schoutens*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*