(5.2) 10 Put each of the five points $(x_1, y_1), \ldots, (x_5, y_5)$ in one of the four boxes

x even	x even	x odd	x odd
y even	y odd	y even	y odd

Then at least one box will contain two points, say (x_i, y_i) and (x_j, y_j), and the midpoint

$$\left(\frac{x_i + x_j}{2}, \frac{y_i + y_j}{2} \right)$$

has integer coordinates.

14 a) Label the boxes 1, 2, 3, 4, 5 and put the seven integers in the five boxes according to
$1, 10 \to$ box 1 ; $2, 9 \to$ box 2 ;
$3, 8 \to$ box 3 ; $4, 7 \to$ box 4 ; $5, 6 \to$ box 5.

There will be at least two boxes that contain two integers.

b) False, e.g. when the six integers are 1, 2, 3, 4, 5, 6.

(5.2) 26 Among 20 people there are either four mutual friends or four mutual enemies.

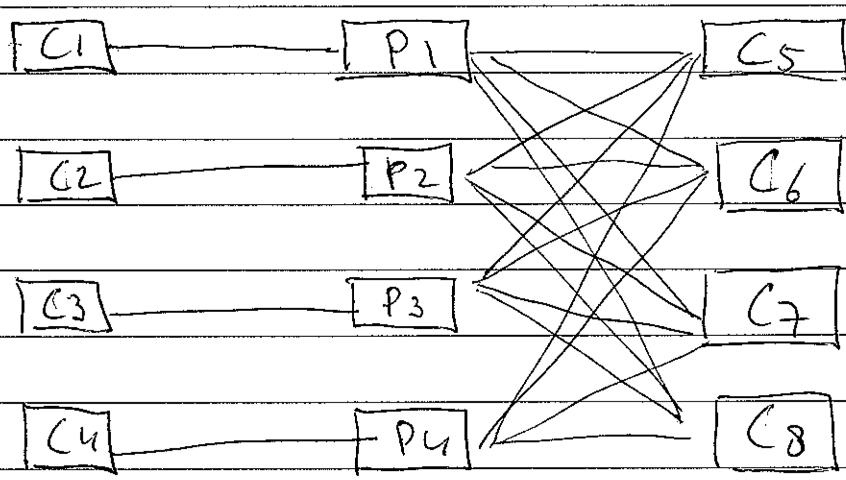
Let A be one of the 20 people. By the pigeonhole principle, A has either at least 10 friends or at least 10 enemies (19 objects, 2 boxes).

A has 10 friends:
Among the 10 friends there are either 4 mutual enemies or 3 mutual friends*. In the second case the 3 mutual friends together with A form 4 mutual friends.

* Exercise 25

A has 10 enemies:
Similar, after replacing friends with enemies.

34



$4 \cdot 1$
$+ \, 4 \cdot 4$
$= 20$

(5.2) <u>34</u> Continued. The given solution with 20 cables is clearly sufficient.

With less then 20 cables, one of the printers would be connected to at most four computers, leaving the remaining four computers with three printers. So the solution with 20 is optimal.

(5.3) <u>16</u>

$$\binom{10}{1} + \binom{10}{3} + \binom{10}{5} + \binom{10}{7} + \binom{10}{9}$$

$$= 10 + 120 + 252 + 120 + 10$$

$$= 512 \quad \Big(= 2^9 = \frac{2^{10}}{2},$$ this is not a coincidence, see Cor. 2 and the remark right after it in section (5.4) p. 365).

(5.4) <u>24</u> $\quad 10! \cdot \binom{11}{6} \cdot 6!$

1st task: order the 10 women

2nd task: choose positions for the 6 men

3rd task: order the 6 men.

Pb 104

$f : \{1, 2, 3, 4, 5, 6\} \longrightarrow \{a, b, c\}$

a) 3^6 choices for f

b) 2^6 choices take values in $\{a, b\}$

2^6 ——— $\{b, c\}$

2^6 ——— $\{a, c\}$

1 choices takes values in $\{a\}$

1 ——— $\{b\}$

1 ——— $\{c\}$

$$N = 3^6 - 3 \cdot 2^6 + 3 \cdot 1 = 540$$