(7.2) 4b

$$\begin{cases} a_n = 7a_{n-1} - 10a_{n-2}, & n \geq 2 \\ a_0 = 2 \quad a_1 = 1 \end{cases}$$

char. equ: $r^2 - 7r + 10 = 0$

$\Leftrightarrow (r-2)(r-5) = 0$

$\Leftrightarrow r_1 = 2 \quad r_2 = 5$

$$a_n = \alpha_1 . 2^n + \alpha_2 . 5^n$$

$$\begin{cases} a_0 = 2 \\ a_1 = 1 \end{cases} \Leftrightarrow \begin{cases} 2 = \alpha_1 + \alpha_2 \\ 1 = 2\alpha_1 + 5\alpha_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} -3 = 3\alpha_2 \\ 1 = 2\alpha_1 + 5\alpha_2 \end{cases}$$

$$\Leftrightarrow \alpha_1 = 3 \quad \alpha_2 = -1$$

$$\boxed{a_n = 3.2^n - 5^n}$$

4f

$$\begin{cases} a_n = -6a_{n-1} - 9a_{n-2}, & n \geq 2 \\ a_0 = 3 \quad a_1 = -3 \end{cases}$$

char. equ.: $r^2 + 6r + 9 = 0$

$\Leftrightarrow (r+3)^2 = 0 \Leftrightarrow r = -3$ (2x)

$$a_n = \alpha_1 . (-3)^n + \alpha_2 . n . (-3)^n$$

$$\begin{cases} a_0 = 3 \\ a_1 = -3 \end{cases} \Leftrightarrow \begin{cases} 3 = \alpha_1 \\ -3 = -3\alpha_1 - 3\alpha_2 \end{cases} \Leftrightarrow \begin{matrix} \alpha_1 = 3 \\ \alpha_2 = -2 \end{matrix}$$

$$\boxed{a_n = 3.(-3)^n - 2n.(-3)^n}$$

(7.2) 4g $\begin{cases} a_{n+2} = -4a_{n+1} + 5a_n, & n \geq 0 \\ a_0 = 2 \quad a_1 = 8 \end{cases}$

$$\left(\Leftrightarrow \begin{cases} a_n = -4a_{n-1} + 5a_{n-2}, & n \geq 2 \\ a_0 = 2 \quad a_1 = 8 \end{cases} \right)$$

Char. equ. : $r^2 + 4r - 5 = 0$

$\Leftrightarrow (r-1)(r+5) = 0$

$\Leftrightarrow r_1 = 1 \quad r_2 = -5$

$$a_n = \alpha_1 + \alpha_2 . (-5)^n$$

$$\begin{cases} a_0 = 2 \\ a_1 = 8 \end{cases} \Leftrightarrow \begin{cases} 2 = \alpha_1 + \alpha_2 \\ 8 = \alpha_1 - 5\alpha_2 \end{cases} \Leftrightarrow \begin{matrix} \alpha_1 = 3 \\ \alpha_2 = -1 \end{matrix}$$

$$\boxed{a_n = 3 - (-5)^n}$$

12 $\begin{cases} a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}, & n \geq 3 \\ a_0 = 3 \quad a_1 = 6 \quad a_2 = 0 \end{cases}$

Char. equ. : $r^3 - 2r^2 - r + 2 = 0$

$\Leftrightarrow (r-2)(r^2 - 1) = 0$

$\Leftrightarrow (r-2)(r+1)(r-1) = 0$

$\Leftrightarrow r_1 = 2, \ r_2 = -1, \ r_3 = 1$

$$a_n = \alpha_1 . 2^n + \alpha_2 (-1)^n + \alpha_3$$

(7.2) 12 cont. $\begin{cases} a_0 = 3 \\ a_1 = 6 \\ a_2 = 0 \end{cases} \iff \begin{cases} 3 = \alpha_1 + \alpha_2 + \alpha_3 \\ 6 = 2\alpha_1 - \alpha_2 + \alpha_3 \\ 0 = 4\alpha_1 + \alpha_2 + \alpha_3 \end{cases}$ -2 -4

$$\iff \begin{cases} 3 = \alpha_1 + \alpha_2 + \alpha_3 \\ 0 = \quad -3\alpha_2 - \alpha_3 \\ -12 = \quad -3\alpha_2 - 3\alpha_3 \end{cases}$$

$$\iff \begin{cases} 3 = \alpha_1 + \alpha_2 + \alpha_3 \\ 0 = \quad -3\alpha_2 - \alpha_3 \\ -12 = \quad -2\alpha_3 \end{cases}$$

$$\iff \alpha_3 = 6 \quad \alpha_2 = -2 \quad \alpha_1 = -1$$

$$\boxed{a_n = -(2^n) - 2 \cdot (-1)^n + 6}$$

18 $\begin{cases} a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} \\ a_0 = -5 \quad a_1 = 4 \quad a_2 = 88 \end{cases}$

char. eqn. : $r^3 - 6r^2 + 12r - 8 = 0$

$\iff (r-2)(r^2 - 4r + 4) = 0$

$\iff (r-2)^3 = 0 \iff r = 2$

(3x)

$$a_n = \alpha_1 . 2^n + \alpha_2 . n . 2^n + \alpha_3 . n^2 . 2^n$$

$$\begin{cases} a_0 = -5 \\ a_1 = 4 \\ a_2 = 88 \end{cases} \iff \begin{cases} -5 = \alpha_1 \\ 4 = 2\alpha_1 + 2\alpha_2 + 2\alpha_3 \\ 88 = 4\alpha_1 + 8\alpha_2 + 16\alpha_3 \end{cases}$$

7.2) 18 cont. $\iff \begin{cases} \alpha_1 = -5 \\ 2\alpha_2 + 2\alpha_3 = 14 \\ 8\alpha_2 + 16\alpha_3 = 108 \end{cases} \iff \begin{cases} \alpha_1 = -5 \\ \alpha_2 = \frac{1}{2} \\ \alpha_3 = \frac{13}{2} \end{cases}$

$$\boxed{a_n = \left(-5 + \frac{1}{2} \cdot n + \frac{13}{2} \cdot n^2\right) \cdot 2^n}$$

24 $\quad a_n = 2a_{n-1} + 2^n$

a) $a_n = n \cdot 2^n$: $\quad n \cdot 2^n \overset{?}{=} 2 \cdot (n-1) \cdot 2^{n-1} + 2^n$ yes

b) $a_n = a_n^{(h)} + a_n^{(p)} = \alpha_1 \cdot 2^n + n \cdot 2^n$

c) $a_0 = 2 \iff 2 = \alpha_1$

$$\boxed{a_n = (2 + n) \cdot 2^n}$$

28 a) $a_n = 2a_{n-1} + 2n^2$, $\quad n \geq 1$

Char. equ. : $r - 2 = 0$

$a_n^{(p)} = an^2 + bn + c$

$$\boxed{a_n = 13 \cdot 2^n - (2n^2 + 8n + 12)}$$

$\begin{cases} n = 1 : & a + b + c = 2c + 2 \\ n = 2 : & 4a + 2b + c = 2a + 2b + 2c + 8 \\ n = 3 : & 9a + 3b + c = 8a + 4b + 2c + 18 \end{cases} \iff \begin{cases} a = -2 \\ b = -8 \\ c = -12 \end{cases}$

$a_n = a_n^{(h)} + a_n^{(p)} = \alpha_1 \cdot 2^n + (-2n^2 - 8n - 12)$

b) $a_1 = 4 \iff 4 = 2\alpha_1 - 22 \iff \alpha_1 = 13$