MATH 225 REVIEW PROBLEMS FOR MIDTERM 2

Details: Midterm 2 will take place in class on Thursday Oct. 23. It will test the material that we covered in class from sections 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, and 4.1.

Suggested problems from the Supplementary Exercises for Chapter 2 on pages 183-184:

1, 2, 7, 9, 10.

Suggested problems from the Supplementary Exercises for Chapter 3 on pages 211-212:

1, 3, 4, 6, 9, 11, 14.

Extra Practice Problems:

(1) Define what it means for a matrix A to be invertible and prove that the inverse of an invertible matrix is unique.

(2) Find the inverse, if it exists, of the following matrices

$$(a) \begin{bmatrix} 5 & 7 \\ 5 & 3 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (c) \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 5 & 6 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Prove that if A is invertible then the columns of A^{-1} are linearly independent.

(4) Suppose that A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$. Show that, in fact, $Ax = b$ has **exactly** one solution for each b in $\mathbb{R}^n$.

(5) Compute the determinants of the following matrices

$$(a) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(6) (a) Describe what happens to $\det A$ when an elementary row or column operation is applied to A.

(b) Suppose A is the $n \times n$ matrix with columns $a_1, a_2, \ldots a_n$, that is

$$A = [a_1 \ a_2 \ \ldots \ a_n].$$

If $\det A = 2$, use your answer from part (a) to compute the following determinants:

(i) $\det[a_2 \ 3a_1 \ a_3 \ \ldots \ a_n]$

(ii) $\det[a_n \ a_1 \ a_2 \ \ldots \ a_{n-1}]$

(iii) $\det[a_1 \ (a_1 - a_2) \ (a_2 - a_3) \ \ldots \ (a_n - a_{n-1})]$

(7) Using elementary row operations to simplify your calculation, compute

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ -3 & 2 & -5 & 13 \\ 1 & -2 & 10 & 4 \\ -2 & 9 & -8 & 25 \end{vmatrix}$$

(8) State Cramer's rule and use it to solve $Ax = b$ for

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(9) (a) Define what it means for V_1 to be a subspace of a vector space V.

(b) Show that if $v_1, v_2, \ldots, v_p$ are elements of a vector space V, then $Span\{v_1, v_2, \ldots, v_p\}$ is a subspace of V.