

MATH 225 REVIEW PROBLEMS FOR MIDTERM 3

Details: Midterm 3 will take place in class on Thursday Nov. 20. It will test the material that we covered in class from sections 4.2, 4.3, 4.5, 4.6, 5.1, 5.2, and 5.3.

Suggested problems from the Supplementary Exercises for Chapter 4 on pages 298-300:

1, 2, 4, 7, 10.

Suggested problems from the Supplementary Exercises for Chapter 5 on pages 370-372:

1 a-r, 2, 3, 6 a, 11, 18.

Extra Practice Problems:

- (1) Define the column space $ColA$ and the nullspace $NulA$ of an $m \times n$ matrix A . Using only these definitions, prove that these are both subspaces of some Euclidean space.

(2) Let $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$.

- (a) Describe $NulA$ as a set of vectors. Find a basis for $NulA$.
(b) Find a basis for $ColA$.
(c) Find the rank of A and the dimension of $NulA$.
- (3) Show that the polynomials 1 , $2t$, $-2 + 4t^2$, and $-12t + 8t^3$ form a basis of \mathbb{P}^3 .
- (4) (a) Determine whether or not the following set of vectors is linearly independent

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

(b) Find a basis for

$$\text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

(5) Find the eigenvalues of $A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$ and show that A is not diagonalizable.

(6) Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$.

- (a) Find the eigenvalues and eigenvectors of A .
- (b) Diagonalize A .
- (c) Compute A^{27} .

(7) Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$.

- (a) Find the characteristic polynomial and the eigenvalues of A .
- (b) Find the set of eigenvectors for each eigenvalue of A . Express these sets as the span of a set of vectors.
- (c) Determine if A diagonalizable.