

MATH 225 REVIEW PROBLEMS FOR CHAPTER 6

Suggested problems from the text:

Section 6.1 ; 1–8, 13–18, 19, 20, 23, 24.

Section 6.2 ; 7–16, 23, 24 (a) (d), 26.

Section 6.3 ; 3–6, 9–12, 15, 16.

Section 6.5 ; 1–6, 7–9, 11, 13, 19, 21, 23, 25–28.

Extra Practice Problems:

- (1) Let u and v be vectors in \mathbb{R}^n . Show that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ if and only if $u \cdot v = 0$.
- (2) (a) Define $proj_v y$, the projection of the vector y onto the line determined by v .
(b) Show that $proj_v y$ is the vector on the line determined by v which is closest to y .

(c) Compute $proj_v y$ for $y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$.

- (3) (a) Define what it means for a set of vectors $\{v_1, v_2, \dots, v_p\}$ to be orthogonal.
(b) Show that if the set $\{v_1, v_2, \dots, v_p\}$ is orthogonal then it is also linearly independent.
- (4) (a) Define $proj_W y$, the projection of the vector y onto the subspace W .
(b) Show that $proj_W y$ is the vector in W which is closest to y .

(c) Compute $proj_W y$ for $y = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ and

$$W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\}.$$

(d) For y and W as in part (c), find the minimum value of $\|y - w\|$ for all w in W .

(5) Find a least squares solution for the following linear systems:

(a)

$$x_1 + x_2 = 4$$

$$x_1 = 1$$

$$x_2 = 1$$

(b)

$$x_1 + x_2 = 2$$

$$x_1 - x_2 = 2$$

$$x_1 + 2x_2 = 1.$$