Math 241 Worksheet 1
Parameterizing Curves and Surfaces

In order to directly compute integrals of the form

$$\int_C f(x,y,z)\,ds, \quad \int_C f(x,y,z)\,dx, \quad \iint_S f(x,y,z)\,dS, \quad \text{or} \quad \iint_S f(x,y,z)\,dx\,dy,$$

you need to know how to parameterize the curves C and the surfaces S. Here are some exercises to test your parameterization skills. Remember to specify the domain of your parameters.

CURVES

(1) Parameterize the curve which is the right half of the circle $x^2 + y^2 = 3$ and is oriented to run from the bottom of the circle to the top.
(2) Parameterize the ellipse $x^2/a^2 + y^2/b^2 = 1$ oriented in the counterclockwise direction.
(3) Parameterize the circle of radius 9 with center at $(4, 7)$ and oriented in the clockwise direction.
(4) Parameterize straight line running from the point $P(1, 2, 3)$ to the point $Q(5, 4, 3)$. Then parameterize the line from Q to P.
(5) Parameterize the ellipse C formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y + 3$ which is oriented in the counterclockwise direction when viewed from above.

SURFACES

(1) Parameterize the sphere of radius 4 with center at $(1, 2, 3)$.
(2) Consider the sphere of radius 1 and center at the origin $(0, 0, 0)$. Pararmeterize the portion of the sphere which lies in side the cone $z = \sqrt{x^2 + y^2}$. Parameterize the portion of the sphere which lies outside of this cone.
(3) Parameterize the part of the plane $2x + 3y + z = 6$ which lies inside of the first octant.
(4) Parameterize the surface defined by the equation $z^2 + y^2 - x = 6$.
(5) Consider the circle in the xz-plane with radius 1 and center at $(7, 0)$. Rotate this circle around the z-axis to obtain a torus (the outside of a donut). Parameterize the torus.