Math 481 Introduction to Differential Geometry

Assignment 2. Due Tuesday Feb. 10

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1. The n-dimensional sphere can be described as follows

$$S^n = \{(x_1, x_2, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} \mid x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1\}.$$

   Use the *Regular Value Theorem* to prove that S^n is a smooth manifold.

2. The set of $n \times n$ orthogonal matrices is

$$O(n) = \{n \times n \text{ matrices } A \mid AA^T = I\}.$$

   Here, A^T is the transpose of A and I is the $n \times n$ identity matrix. Use the *Regular Value Theorem* to prove that $O(n)$ is a smooth manifold. What is the dimension of $O(n)$?