

Math 481 Introduction to Differential Geometry

Assignment 7, Due Thursday April 9

1. Let α be a 2-form on \mathbb{R}^3 given by

$$\alpha = 2xy \, dx \wedge dy - (x^2 + y + 1) \, dy \wedge dz.$$

- (a) Compute $d\alpha$.
(b) Let $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the smooth mapping defined by

$$F(u, v) = (u + v, u^2 + 1, 3uv - 4).$$

Compute $F^*\alpha$.

- (c) Let $\beta = dx + 2dy + 3dz$. Compute $\alpha \wedge \beta$ and $F^*(\alpha \wedge \beta)$.

2. Consider the following two vector fields on \mathbb{R}^3 :

$$X = 2xy \frac{\partial}{\partial x} + (x^2 + z^2) \frac{\partial}{\partial z},$$

$$Y = e^{xy} \frac{\partial}{\partial x} + \cos(2xz) \frac{\partial}{\partial y}.$$

Compute the vector field $[X, Y]$.

3. Let α be a 1-form on a 3-dimensional manifold M . Prove that $d(d\alpha) = 0$.
4. Let ω be an r -form on a manifold M with $r \geq 2$. Suppose that the tangent vectors $V_1, \dots, V_r \in T_p M$ are linearly dependent. Prove that

$$\omega(p)(V_1, \dots, V_r) = 0.$$

5. Let X, Y and Z be smooth vector fields on a manifold M . Prove the Jacobi Identity

$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0.$$