

Math 423 Differential Geometry Fall 2006

Homework 1: Curves Due Thursday Sept. 7

1. p.22 #2
2. p.22 #9
3. p.55 #2
4. p.55 #4
5. Consider the following three curves:

$$\alpha(t) = (\sqrt{t^2 + 1}, t, 0), \quad -\infty < t < \infty$$

$$\beta(t) = (\cosh t, \sinh t, 0), \quad -\infty < t < \infty$$

$$\gamma(t) = \left(\frac{1+t^2}{1-t^2}, \frac{2t}{1-t^2}, 0 \right), \quad -1 < t < 1$$

- (a) Show that each of these curves lies on the hyperbolic cylinder $x^2 - y^2 = 1$ in \mathbb{R}^3 .
 - (b) Compute the velocity vector at the point $(\sqrt{2}, 1, 0)$ for each curve.
6. Draw the curve $\alpha(t) = (t, |t|, 1)$. Show that the velocity vector of α at the point $(0, 0, 0)$ does not exist. Reparameterize α so the new curve $\beta(s) = \alpha(h(s))$ has a well-defined velocity vector at $(0, 0, 0)$.
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Bonus Questions for those taking the course for four credits.

7. p.55 #10
8. p.55 #11