

**Math 423    Differential Geometry    Fall 2006**

Homework 4: The Main Theorem of Space Curves

Due Thursday Sept. 28

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1. Section 3.5 #3
  2. Section 3.5 #6
  3. Section 3.5 #7
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**Bonus Questions** for those taking the course for four credits or for those who want more of a challenge.

4. The Frenet frame is not the only useful way to frame a curve. Let  $\alpha: I \rightarrow \mathbb{R}^3$  be a unit speed curve and consider a frame  $(T(t), E_1(t), E_2(t))$  along  $\alpha$  which is positively oriented and satisfies

$$E_1'(t) = -k_1(t)T(t) \quad \text{and} \quad E_2'(t) = -k_2(t)T(t)$$

for some functions  $k_1(t)$  and  $k_2(t)$ .

- (a) Prove that two curves with the same  $k_1(t)$  and  $k_2(t)$  are congruent.
- (b) Assuming  $\kappa(t) > 0$ , express  $k_1(t)$  and  $k_2(t)$  in terms of  $\kappa(t)$  and  $\theta(t)$  where  $\theta(t)$  is defined by the equation

$$N(t) = \cos \theta(t)E_1(t) + \sin \theta(t)E_2(t).$$

- (c) Express  $\kappa$  and  $\tau$  in terms of  $k_1$  and  $k_2$ .
- (d) Show that  $T' = k_1E_1 + k_2E_2$ .
- (e) Show that  $k_1(t)$  and  $k_2(t)$  are constant iff  $\alpha$  is part of a circle. What is the relation of the radius of the circle and  $k_1$  and  $k_2$ ?
- (f) Show that  $\alpha$  is a plane curve iff  $k_1(t)/k_2(t)$  is constant.