Math 423 Differential Geometry Fall 2006

Homework 8: More on Curvature

Due Thursday Nov. 9

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1. Prove that $\nabla_{\bar{x}_u}\nabla_{\bar{x}_v}U = \nabla_{\bar{x}_v}\nabla_{\bar{x}_u}U$
2. Prove that a surface of revolution with $K = 0$ is part of a cone or a plane. Hint: model your proof after our proof that a surface of revolution with $H = 0$ is part of a catenoid or a plane.
3. Compute the Gaussian curvature of the sphere of radius R using the formula from Gauss's Theorem Egregium:

$$K = -\frac{1}{2\sqrt{EG}}\left(\frac{\partial}{\partial v}\left(\frac{E_v}{\sqrt{EG}}\right) + \frac{\partial}{\partial u}\left(\frac{G_u}{\sqrt{EG}}\right)\right).$$

Bonus Questions for those taking the course for four credits or for those who want more of a challenge.

4. Show that the U terms in the equations $x_{uuv} - x_{uvu} = 0$ and $x_{vvu} - x_{uvu} = 0$ give

$$L_v - M_u = M\left(\frac{G_u}{2G} - \frac{E_u}{2E}\right) + E_v H = L\frac{E_v}{2E} + M\left(\frac{G_u}{2G} - \frac{E_u}{2E}\right) + N\frac{E_v}{2G}$$

and

$$N_u - M_v = M\left(\frac{E_v}{2E} - \frac{G_v}{2G}\right) + G_u H = L\frac{G_u}{2E} + M\left(\frac{E_v}{2E} - \frac{G_v}{2G}\right) + N\frac{G_u}{2G}.$$

What do these equations say in the case when $H = 0$ and the parameterization satisfies $E = G$?