Math 423 Differential Geometry

Assignment 1, Due Tuesday Sept 8

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1. Let $c\colon I \to \mathbb{R}^2$ be a regular plane curve. Determine how the curvature of c changes if one applies the following transformations of $\mathbb{R}^2$ to its image:

   (a) a translation $(x_1, x_2) \mapsto (x_1 + C_1, x_2 + C_2)$;

   (b) a rotation $R_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$;

   (c) a reflection $(x_1, x_2) \mapsto (-x_1, x_2)$;

   (d) a dilation $(x_1, x_2) \mapsto (rx_1, rx_2)$.

2. Compute the curvature of the ellipse $\{(x_1, x_2) \mid \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1\}$ where $0 < a < b$. At what point(s) does the curvature take on its maximum and minimum values?

3. (Bonus) Let $c\colon I \to \mathbb{R}^2$ be a regular plane curve such that $\|c(t)\| \leq 1$ for all $t \in I$. Suppose there is a point $t_0 \in I$ such that $\|c(t_0)\| = 1$. Prove that the curvature at that point satisfies $|\kappa(t_0)| \geq 1$.