

# Math 423 Differential Geometry

## Assignment 3, Due Thursday Sept 24

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1. Show that the plane curve  $c(t) = (\sin t, \sin(2t))$  is regular and closed but is not simple. Compute the rotation index of this curve.
2. Let  $c: \mathbb{R} \rightarrow \mathbb{R}^2$  be a closed curve of period 5. Suppose that  $c$  also satisfies

$$c(t+1) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} c(t),$$

for  $\alpha = 2\pi/5$ . Compute the rotation number of  $c$ .

Hint: write  $\frac{\dot{c}(t)}{\|\dot{c}(t)\|} = (\cos \theta(t), \sin \theta(t))$  and interpret the condition above in terms of the function  $\theta(t)$ .

3. (bonus) Let  $c(s)$  and  $d(s)$  be two closed plane curves parameterized by arclength. Suppose that the line segment joining  $c(s)$  to  $d(s)$  does not pass through the origin for all  $s$ . Show that  $c$  and  $d$  have equal rotation numbers.