Math 423 Differential Geometry

Assignment 4, Due Thursday Oct.7

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1. Let $f\colon U \to \mathbb{R}^3$ be a surface element. Suppose that the image of f lies in the region$\{(x, y, z) \mid z > 0\}$, and the tangent plane to f at $u = 0 \in \mathbb{R}^2$ is parallel to the plane $z = 0$. Show that the principal curvatures of f at $f(0)$ satisfy $\kappa_1\kappa_2 \geq 0$.

2. Let $f\colon U \to \mathbb{R}^3$ be a surface element. Define the parallel surface element at distance ϵ by
$$\tilde{f}(s, t) = f(s, t) + \epsilon\nu(s, t).$$
Show that the principal curvatures of f and $\tilde{f}$ are related by the following formula
$$\tilde{\kappa}_i = \frac{\kappa_i}{1 - \epsilon\kappa_i}, \ (i = 1, 2).$$
(You may assume that ϵ is as small us you like.)

3. Let $f\colon U \to \mathbb{R}^3$ be a surface element and let X_1 be a unit eigenvector in $T_u f$ for the first principal curvature κ_1 at $p = f(u)$. Let $X(\theta)$ be the unit vector in $T_u f$ which makes an angle θ with X_1. Prove that the mean curvature of f at p, $H(p)$, satisfies
$$H(p) = \frac{1}{2\pi}\int_0^{2\pi} \kappa_{X(\theta)}\, d\theta.$$
(Recall that $\kappa_X = II(X, X)$.)