Math 423 Differential Geometry

Assignment 5, Due Thursday Oct.22

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1. Let $f\colon U \to \mathbb{R}^3$ be a conformal surface element. In particular, assume that the first fundamental form satisfies $g_{12}(s,t) = 0$, and $g_{11}(s,t) = g_{22}(s,t) = e^{\phi(s,t)}$. Compute the Gauss curvature K.

2. Find a surface element other than the pseudo-sphere which has constant Gauss curvature $K = -1$.

3. Given a surface element $f(s,t)$ set $\tilde{f}(s,t) = f(s,-t)$. How are the Gauss curvatures and mean curvatures of f and $\tilde{f}$ related.

4. Let $f\colon U \to \mathbb{R}^3$ be a surface element. For two vector field X and Y on $\mathbb{R}^3$ define
$$[X,Y] = D_X Y - D_Y X.$$
   (a) If X and Y are tangent to f show that
$$[X,Y] = \nabla_X Y - \nabla_Y X.$$
   (b) Verify that
$$\left[\frac{\partial f}{\partial u^i}, \frac{\partial f}{\partial u^j}\right] = 0, \quad i,j = 1,2.$$
   (c) Use this to prove that $\Gamma_{ij,k} = \Gamma_{ji,k}$.