

# Math 518 Differential Manifolds I

## Assignment 3, Due Thursday Sept 24

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1. (a) Verify that the map

$$t \mapsto \left( \frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2} \right)$$

is an embedding of  $\mathbb{R}$  into  $\mathbb{R}^2$ .

- (b) Determine if the map  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  given by

$$F(x, y) = (x \cos y, x \sin y, x)$$

is an immersion.

- (c) Check that the quotient map  $q: \mathbb{R}^{n+1} \setminus \{0\} \rightarrow \mathbb{R}P^n$  is a submersion.

2. If  $M$  is compact and  $N$  is connected show that every submersion  $F: M \rightarrow N$  is surjective.
3. Let  $M$  be a compact manifold and assume that  $\dim M = \dim N$ . Let  $y \in N$  be a regular value of a smooth mapping  $F: M \rightarrow N$ . Show that  $F^{-1}(y)$  is a finite set  $\{x_1, \dots, x_k\}$ . Prove that there exists a neighborhood  $V$  of  $y$  in  $N$  such that  $F^{-1}(V)$  is a disjoint union  $U_1 \cup \dots \cup U_k$  where each  $U_j$  is an open neighborhood of  $x_j$  and  $F|_{U_j}$  is a diffeomorphism onto  $V$ .
4. Show that the image of  $S^2 \subset \mathbb{R}^3$  under the map

$$F: (x, y, z) \mapsto (x^2, y^2, z^2, \sqrt{2}yz, \sqrt{2}zx, \sqrt{2}xy)$$

is a submanifold of  $\mathbb{R}^6$ . Hint: consider  $F \circ q$  for the quotient map  $q: \mathbb{R}^3 \setminus \{0\} \rightarrow \mathbb{R}P^2$ .