Lecture 11 Math 225

Note Title 2/21/2008

Summary

— Discuss Midterm

Sect 2.3 Characterization of Invertible Matrices

— Review Chapter 2

Section 2.3 Characterization of invertible Matrices

Theorem 8 Let A be a square $n \times n$ matrix. Then the following are equivalent (the statements are either all true or all false):

a. A is invertible
b. A is row equivalent to I_n
c. A has pivot position in each column
d. $Ax = 0$ has only the trivial solution
e. The columns of A form a linearly independent set of vectors

g'. A has a pivot position in each row
g. The equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$.
h. The columns of A span $\mathbb{R}^n$.
j. There exists $n \times n$ matrix C such that $CA = I_n$
k. There exists $n \times n$ matrix D such that $AD = I_n$
l. A^T is invertible
m. The rows of A form a linearly independent set of vectors
n. The rows of A span $\mathbb{R}^n$.

<u>Proof</u> $a \Leftrightarrow b$. See Theorem 7 Section 2.2.

$b \Leftrightarrow c$ $\begin{cases} b \Rightarrow c. \text{ Obvious} \\ c \Rightarrow b. \end{cases}$ If A is square and has pivot position in each column then these positions are on the diagonal $\Rightarrow A \sim I_n$.

$c \Leftrightarrow d$ See section 1.5

$d \Leftrightarrow e$ See section 1.7

$g' \Leftrightarrow b$ $\begin{cases} b \Rightarrow g' \text{ Obvious} \\ g' \Rightarrow b \text{ Similar to } c \Rightarrow b \end{cases}$

$g' \Leftrightarrow g \Leftrightarrow h$ See section 1.4 Theorem 4

$a \Leftrightarrow j$ and $a \Leftrightarrow k$ see Solution to problem 6 in Midterm 1.

$a \Leftrightarrow l$ $\begin{cases} a \Rightarrow l & \text{See Section 2.2 Theorem 6} \\ l \Rightarrow a & \text{If } A^T \text{ is invertible then} \end{cases}$

by Theorem 6 Section 2.2 $(A^T)^T$ is invertible but $(A^T)^T = A$ by Theorem 3 Section 2.1.

$a \Leftrightarrow m$

$a \Rightarrow m$ If A is invertible then A^T is invertible then the columns of A^T are linearly independent $\Rightarrow$ the rows of A are linearly independent.

$m \Rightarrow a$ If the rows of A are linearly independent then the columns of A^T are linearly independent $\Rightarrow$ A^T is invertible $\Rightarrow$ A is invertible

$a \Leftrightarrow n$ Homework!

Review Chapter 2

Product of matrices
- definition
- methods of calculation (2)
- properties

Transpose of a matrix
- definition
- methods of calculation (2)
- properties

Inverse of a $n \times n$ matrix A
- definition
- methods of calculation
  - for 2×2
  - for $n \times n$
- characterization
  - $A \sim I_n$
  - $Ax = 0$ has unique sln
  - $Ax = b$ has at least one sln for any b in $\mathbb{R}^n$

Elementary matrices
- definition
- relation with row operations
- are they invertible?